MOSEK Rmosek Package

Release 8.1.0.15(beta)

MOSEK ApS

2017

INTRODUCTION

The **MOSEK** Optimization Suite 8.1.0.15(beta) is a powerful software package capable of solving large-scale optimization problems of the following kind:

- linear,

- conic quadratic (also known as second-order cone),

- convex quadratic,

- semidefinite,

- and general convex.

Integer constrained variables are supported for all problem classes except for semidefinite and general convex problems. In order to obtain an overview of features in the **MOSEK** Optimization Suite consult the product introduction guide.

The most widespread class of optimization problems is *linear optimization problems*, where all relations are linear. The tremendous success of both applications and theory of linear optimization can be ascribed to the following factors:

- The required data are simple, i.e. just matrices and vectors.

- Convexity is guaranteed since the problem is convex by construction.

- Linear functions are trivially differentiable.

- There exist very efficient algorithms and software for solving linear problems.

- Duality properties for linear optimization are nice and simple.

Even if the linear optimization model is only an approximation to the true problem at hand, the advantages of linear optimization may outweigh the disadvantages. In some cases, however, the problem formulation is inherently nonlinear and a linear approximation is either intractable or inadequate. *Conic optimization* has proved to be a very expressive and powerful way to introduce nonlinearities, while preserving all the nice properties of linear optimization listed above.

The fundamental expression in linear optimization is a linear expression of the form

$$Ax - b \in \mathcal{K}$$

where $\mathcal{K} = \{y : y \geq 0\}$, i.e.,

$$Ax - b = y,$$
$$y \in \mathcal{K}.$$

In conic optimization a wider class of convex sets $\mathcal{K}$ is allowed, for example in 3 dimensions $\mathcal{K}$ may correspond to an ice cream cone. The conic optimizer in **MOSEK** supports three structurally different types of cones $\mathcal{K}$, which allows a surprisingly large number of nonlinear relations to be modelled (as described in the **MOSEK** modeling cookbook), while preserving the nice algorithmic and theoretical properties of linear optimization.

1.1 Why the Rmosek Package?

The Rmosek Package provides access to most functionalities of **MOSEK** from an R-language software environment. The package is adjusted for the typical R user.

The Rmosek Package provides access to:

- Linear Optimization (LO)
- Conic Quadratic (Second-Order Cone) Optimization (CQO, SOCO)
- Convex Quadratic and Quadratically Constrained Optimization (QCQO)
- Semidefinite Optimization (SDO)
- Separable Convex Optimization (SCO)

TWO

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You can get in touch with **MOSEK** using popular social media as well:

Blogger	http://blog.mosek.com/
Google Group	https://groups.google.com/forum/#!forum/mosek
Twitter	https://twitter.com/mosektw
Google+	https://plus.google.com/+Mosek/posts
Linkedin	https://www.linkedin.com/company/mosek-aps

In particular **Twitter** is used for news, updates and release announcements.

LICENSE AGREEMENT

Before using the **MOSEK** software, please read the license agreement available in the distribution at `<MSKHOME>/mosek/8/mosek-eula.pdf` or on the **MOSEK** website https://mosek.com/sales/license-agreement.

MOSEK uses some third-party open-source libraries. Their license details follows.

zlib

MOSEK includes the *zlib* library obtained from the zlib website. The license agreement for *zlib* is shown in Listing 3.1.

Listing 3.1: *zlib* license.

```
zlib.h -- interface of the 'zlib' general purpose compression library
version 1.2.7, May 2nd, 2012

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Jean-loup Gailly        Mark Adler
jloup@gzip.org          madler@alumni.caltech.edu
```

fplib

MOSEK includes the floating point formatting library developed by David M. Gay obtained from the netlib website. The license agreement for *fplib* is shown in Listing 3.2.

Listing 3.2: *fplib* license.

```
/****************************************************************
 *
```

```
 * The author of this software is David M. Gay.
 *
 * Copyright (c) 1991, 2000, 2001 by Lucent Technologies.
 *
 * Permission to use, copy, modify, and distribute this software for any
 * purpose without fee is hereby granted, provided that this entire notice
 * is included in all copies of any software which is or includes a copy
 * or modification of this software and in all copies of the supporting
 * documentation for such software.
 *
 * THIS SOFTWARE IS BEING PROVIDED "AS IS", WITHOUT ANY EXPRESS OR IMPLIED
 * WARRANTY.  IN PARTICULAR, NEITHER THE AUTHOR NOR LUCENT MAKES ANY
 * REPRESENTATION OR WARRANTY OF ANY KIND CONCERNING THE MERCHANTABILITY
 * OF THIS SOFTWARE OR ITS FITNESS FOR ANY PARTICULAR PURPOSE.
 *
 *****************************************************************/
```

INSTALLATION

In this section we discuss how to install and setup the **MOSEK** Rmosek Package.

Important: Before running this **MOSEK** interface please make sure that you:

- Installed **MOSEK** correctly. Some operating systems require extra steps. See the Installation guide for instructions and common troubleshooting tips.

- Set up a license. See the Licensing guide for instructions.

4.1 Windows Platforms

The interface documented here is part of the Rmosek package distributed from http://rmosek.r-forge. r-project.org/. Notice, however, that a pre-compiled binary version of it has not been distributed. Thus a small amount of configuration is necessary in order to install the package. It is not very difficult, but besides the basic installation of R you will need two pieces of software readily available through the Internet. In summary:

- **MOSEK** (the optimization library we interface to.)

- Rtools (the tools needed for R package development

The following is a step-by-step guide through the installation of the package.

For additional help installing this interface, the section on packages in the R for Windows *FAQ* available on the CRAN website may be useful. The *R Installation and Administration* manual, also published by CRAN on their website, is another good source of information.

4.1.1 Setting Up the Target Machine

Assuming a working installation of R and **MOSEK** on the machine targeted for the R-to-**MOSEK** interface, the first step is to download Rtools for Windows. From this program you will need to install the component called *R toolset*, the *Cygwin DLLs*, and the R toolchain.

Note: R toolset and Cygwin DLLs will extend the Windows CMD with Unix-style commands, while the R toolchain (based on MinGW compilers) makes it possible to compile the C++ source code in the package. These components can be replaced by any other Unix-style shell and C++ build chain, but in the remaining guide the use of Rtools will be assumed.

After the installation of Rtools you will have to set the Windowsenvironment variable called `PATH`, in order to utilize the components. Assuming that the home directory of the Rtools installation was `C:\Rtools`, the entries shown below will have to be added to the existing `PATH` variable. Note that all entries in the `PATH` variable must be separated by a semicolon (;), and that all these entries have to represent folders that exist on the target machine.

- Add `C:\Rtools\bin;` to enable the R toolset and Cygwin DLLs.

- Add `C:\Rtools\gcc-`*`VERSION`*`bin;` (for some *VERSION*) to enable the R toolchain.

That was it, but before we move on to the installation process, please ensure that the PATH variable also contains the `bin` folder of all **MOSEK** installations (32 and/or 64 bit) that you wish this interface to target. This is necessary for automatic configuration to work, and could look something like:

```
C:\Program\Mosek\8\tools\platform\win64x86\bin
```

4.1.2 Installing the Package with Automatic Configuration

Automatic configuration works equivalently to calling `where mosek` in the Windows CMD. It searches the environment variable called `PATH` for a folder with an executable called `mosek`. Note that if more than one such folder exists, only the one mentioned first in the `PATH` variable is chosen. It then determines the most ordinary of the available optimization libraries within this folder (typically `mosek.lib` or `mosek64.lib`), along with other relevant information. This configuration should work for all users installing the package on a single architecture (64 or 32 bit) and only requiring the ordinary optimization library. Otherwise, manual configuration of the package will be needed.

Now open R for the architecture (64 or 32 bit) you wish to install the package on. Make sure all your packages are up to date by writing `update.packages()`, and execute a command similar to the one shown below. This will install the Rmosek package:

```
install.packages("Rmosek", type="source", INSTALL_opts="--no-multiarch", repos="http://
↪download.mosek.com/R/8")
```

Note that this package will have to be installed from *source* as it needs a static link to the **MOSEK** optimization library. This unfortunately means that dependencies (i.e. the *Matrix* package) will also be installed from source if new releases are available. Since it is more time-consuming to install the *Matrix* package from source, it is recommended to start with a call to `update.packages()`. The availability of the *Matrix* package should not be a problem, as it has been part of the standard R installation since version 2.9.0.

4.1.3 Installing the Package with Manual Configuration

If the automatic configuration does not suit your particular needs, or fails for some reason, a manual configuration may work instead. Unfortunately the `configure.vars` parameter of the `install.packages` command does not work on Windows, meaning that the files of the Rmosek source package will have to be edited. This can be difficult for non-savvy users, but have hopefully been documented sufficiently here.

How to manually configure the Rmosek package?

When you download the package from http://rmosek.r-forge.r-project.org/, it comes in a compressed archive called `RMOSEK_`*`VERSION`*`.tar.gz` for some version number `VERSION`. In order to configure the package, you will have to go through the following steps. A more thorough explanation of each step will be given afterwards.

1. Extract the archive into a directory.

2. Setup the local system descriptors, `Localsys.txt`, for each of the sub-architectures you will be using: 64 bit (e.g. x64) and/or 32 bit (e.g. i386). These files come with a guide written into them that clearly states how this should be done.

3. Compress the directory back into an archive.

The first step is to extract the `rmosek_`*`VERSION`*`.tar.gz` archive that you downloaded. This can either be done by using one of the many tools freely available online, or the `tar` command that was installed

with Rtools. If you choose to use the `tar` command, you can extract the package by opening Windows CMD and executing a command similar to:

```
tar --no-same-owner -zxvf LOCATION/RMOSEK_VERSION.tar.gz
```

Remember to exchange *LOCATION* and *VERSION* with the correct values. For those not familiar with Windows CMD, we recommend the use of an external tool as this configuration can then be performed entirely within Windows Explorer.

The second step is to tell the package where to find the **MOSEK** optimization library. Open the extracted directory called `Rmosek` in either Windows CMD or Windows Explorer, and navigate to the subdirectory called `src\setup`. If you want to install a 64 bit version of the package (making an interface between 64 bit R and 64 bit **MOSEK**), open the folder `x64` and follow the guide in `Localsys.txt`. If you want to install a 32 bit version of the package (making an interface between 32 bit R and 32 bit **MOSEK**), open instead the folder `i386` and follow the guide in this `Localsys.txt`. An example of this is shown below

Listing 4.1: An example of local system descriptor.

```
##
## Greetings user of the R-to -MOSEK interface !
##
## If you are sitting on a WINDOWS 64 bit platform , this is the file that
## you will have to setup before this package can be installed .
## (see e.g. the R-to -MOSEK userguide )
##
#################
## Step 1 of 2 ##
#################
## Please substitute [ MOSEK_HOME_PATH ] below , with the path to the
## platform - specific folder within the MOSEK installation you want to
## use. Note that this path should contain a "bin" and a "h" folder .
## ------------------------------------------------------------------
## For example you can write:
## PKG_MOSEKHOME =C:\ Progra ~1\ Mosek \8\ tools\ platform \ win64x86
##
## If your computer contains the two directories :
## C:\ Progra ~1\ Mosek \8\ tools\ platform \ win64x86 \bin
## C:\ Progra ~1\ Mosek \8\ tools\ platform \ win64x86 \h
## ------------------------------------------------------------------
PKG_MOSEKHOME =[ MOSEK_HOME_PATH ]
#################
## Step 2 of 2 ##
#################
## Please substitute [ MOSEK_LIB_FILE ] below , with the name of the library
## you wish to use within the "bin" folder of your PKG_MOSEKHOME path.
## This "bin" folder must contain a file called [ MOSEK_LIB_FILE ]. lib.
## ------------------------------------------------------------------
## Continuing the example from above , you can write:
## PKG_MOSEKLIB = mosek64_8_0
##
## If your computer contains the file:
## C:\ Progra ~1\ Mosek \8\ tools\ platform \ win64x86 \bin\ mosek64_8_0 .lib
## ------------------------------------------------------------------
PKG_MOSEKLIB =[ MOSEK_LIB_FILE ]
```

The third and final step is to compress the (previously extracted and now altered) directory called `Rmosek`, back into the `Rmosek_VERSION.tar.gz` archive. Again you can either make use of the external tools from step one, or open the Windows CMD and execute a command similar to:

```
tar -zcvf Rmosek_VERSION.tar.gz Rmosek`
```

How to install the manually configured Rmosek package?

Open R, either the console or the graphical user interface, for the architecture (64 or 32 bit) you wish to install the package on. Make sure all your packages are up to date by writing `update.packages()`, and execute the following command to install the Rmosek package::

```
install.packages("LOCATION/Rmosek_VERSION.tar.gz", repos=NULL, type="source", INSTALL_opts="--
↪no-multiarch")
```

Remember to exchange *LOCATION* and *VERSION* with the correct values. The first argument should be the path to your manually configured package. The second argument tells that the package is local and not in an online repository. The third argument tells that it is a source package and so should be compiled. The fourth and final argument specifies that you only wish to install the package to the specific architecture (64 or 32 bit) of the opened R program. If you wish to install on both architectures, and did configure the package for both 32 and 64 bit, simply remove this last argument.

Notice that if you wish to uninstall the Rmosek package at some point, this can be done as for any other package with the command `remove.packages(Rmosek)`.

4.2 UNIX-alike Platforms

The interface documented here is part of the Rmosek package distributed from http://rmosek.r-forge.r-project.org/. Notice, however, that a pre-compiled binary version of it has not been distributed (Such a binary would have to be built individually for each version of R and **MOSEK**). Thus a small amount of configuration is necessary in order to install the package. It is not very difficult, but besides the basic installation of R you will need **MOSEK** (the optimization library we interface to) readily available through the Internet.

The following is a step-by-step guide through the installation of the package.

For additional help on installing this interface, the section on installing packages in the manual *R Installation and Administration*, published on the CRAN website, is a good source of information.

4.2.1 Setting Up the Target Machine

We assume here that you have a working installation of R and **MOSEK** on the machine targeted for the R-to-**MOSEK** interface. The architectures (32 or 64 bit) of these two programs must be exactly the same for consistency. We further assume that the target machine have a build chain (e.g. `GCC` compilers for C/C++) installed.

For automatic configuration to work, the `PATH` variable should contain the `bin` folder of the single **MOSEK** installation (32 or 64 bit) that you wish this interface to target. This could look something like:

```
~/mosek/8/tools/platform/linux64x86/bin
```

If more than one `bin` folder from a **MOSEK** installation are specified, only the first one will be found by automatic configuration. Additional architectures can be added afterwards if necessary.

4.2.2 Installing the Package with Automatic Configuration

Automatic configuration works equivalently to calling `which mosek` in a terminal window. It searches the environment variable called `PATH` for a folder with an executable called `mosek`. Note that if more than one such folder exists, only the one mentioned first in the `PATH` variable is chosen. It then determines the most ordinary of the available optimization libraries within this folder (typically `libmosek` or `libmosek64` with the extension `.so` or `.dylib`), along with other relevant information. This configuration should

work for all users only installing the package on a single architecture (64 or 32 bit) and only requiring the ordinary optimization library. Otherwise, manual configuration of the package will be needed.

Now open R for the architecture (64 or 32 bit) you wish to install the package on. Make sure all your packages are up to date by writing `update.packages()`, and execute a command similar to the one shown below. This will install the Rmosek package::

```
install.packages("Rmosek", type="source", INSTALL_opts="--no-multiarch", repos="http://
↪download.mosek.com/R/8")
```

Note that this package will have to be installed from *source* as it needs a static link to the **MOSEK** optimization library. This unfortunately means that dependencies (i.e. the 'Matrix' package) will also be installed from source if new releases are available. Since it is more time-consuming to install the *Matrix* package from source, it is recommended to start with a call to `update.packages()`. The availability of the *Matrix* package should not be a problem, as it has been part of the standard R installation since version 2.9.0.

4.2.3 Installing the Package with Manual Configuration

Open R for one of the architectures (64 or 32 bit) you wish to install the package on. Make sure all your packages are up to date by writing `update.packages()`, and execute a command similar to the one shown below, with a correct definition of `PKG_MOSEKHOME` and `PKG_MOSEKLIB`. This will install the Rmosek package:

Remember to exchange the `...` of both `PKG_MOSEKHOME` and `PKG_MOSEKLIB` with the correct values, explained as follows. The definition of the first argument, `PKG_MOSEKHOME`, should be the folder in your **MOSEK** installation, containing a `bin` and `h` subdirectory for the platform and architecture matching that of the opened R program. This could for instance look something like:

```
/home/username/mosek/8/tools/platform/linux64x86
```

Notice that auto-expansions such as ∼ does not work, and in case the folder definition contains spaces you will either have to wrap the definition in single-quotes or add backslashes in front of all spaces.

The definition of the argument `PKG_MOSEKLIB` should be the name of the optimization library in the `bin` subdirectory that you wish to utilize in the Rmosek package. This library will be statically linked to the package after a successful installation. Note that the name of the optimization library should be specified without the *lib* prefix, and without its file-extension. The `PKG_RMOSEKLIB` would thus normally be either `mosek` or `mosek64` (linking to respectively `libmosek.so` and `libmosek64.so`, or respectively `libmosek.dylib` and `libmosek64.dylib`, depending on the Unix-alike system). Using `mosek64` requires a 64 bit version of the opened R program and **MOSEK** installation, while `mosek` implies 32 bit.

How to install on multiple architectures?

If you wish to install the Rmosek package on multiple sub-architectures, you will first have to follow the above guide and install the package on one of the architectures (e.g. 32 or 64 bit). Afterwards as explained here, it can then be extended to other sub-architectures. This is also explained in *R Installation and Administration* (published on the CRAN website), in the subsection *Multiple sub-architectures* under *Installing packages*. For this to work you will need a preinstalled version of R and **MOSEK** in all the sub-architectures you wish this package to work with.

Open R in the sub-architecture you wish to extend your installation to, and execute a command similar to the one shown below::

```
install.packages("Rmosek", type="source", libs_only=TRUE, repos="http://download.mosek.com/R/8
↪", configure.vars="PKG_MOSEKHOME=... PKG_MOSEKLIB=...")
```

Remember to exchange ... of the variables `PKG_MOSEKHOME` and `PKG_RMOSELIB`, so that declared **MOSEK** installation always correspond to the current sub-architecture of the opened R program.

How to install from an offline location?

This is almost the same a described above except that you would have to add the argument `repos=NULL` to tell R that it should not use an online repository. Also, instead of writing the package name Rmosek, you should write the entire file-location of the package source file which should look something like `LOCATION/Rmosek_`*`VERSION`*`.tar.gz`.

What are the command-line equivalents?

Sometimes you do not want to open R for all sub-architectures, but instead perform the installation directly from the console. Notice though, that this will require you to download the package source as you will only be able to install from an offline location in this way. In the two commands below you will have to replace `SUBARCH_PATH`, `DIR` and `VERSION` with the correct values, but doing so should be straight forward. Notice that on newer versions of R ($>=2.12$), you can also use the call `R --arch SUBARCH`, instead of specifying the `SUBARCH_PATH`. Remember to exchange `...` of the variables `PKG_MOSEKHOME` and `PKG_MOSEKLIB`, with the correct values as previously explained.

For the first architecture::

```
SUBARCH_PATH/R CMD INSTALL DIR/Rmosek_VERSION.tar.gz --no-multiarch --configure-vars="PKG_
↪MOSEKHOME=... PKG_MOSEKLIB=..."
```

For subsequent architectures::

```
SUBARCH_PATH/R CMD INSTALL DIR/Rmosek_VERSION.tar.gz --libs-only --configure-vars="PKG_
↪MOSEKHOME=... PKG_MOSEKLIB=..."
```

GUIDELINES

5.1 Parallel optimization Using the The Multicore Package

The R package called *multicore*, provides functions for parallel execution of R code on machines with multiple cores or CPUs. Windows is not currently supported, but the package should work on most UNIX-alike platforms.

The multicore package works by copying the full memory state of the R session to new processes. While this seems like a large overhead, in practice, the copy is delayed until modification assuring a smooth parallel execution. The downside is that this low-level memory state copy is not safe for all types of resources. As an example, parallel interactions with the GUI or on-screen devices can cause the R session to crash. It is thus recommended only to use the multicore package in console R.

In the Rmosek package a license is an externally acquired resource, and attempts to simply copy the memory state of this resource will provoke a session crash. Thus, licenses should always be released before the time of parallelization.

Note: Always call `mosek_clean()` before a parallelizing operator. Failure to do so is likely to provoke session crashes.

A consequence of this is that each new process will be using a separate license. That is, your license system should allow 8 licenses to be checked out simultaneously, if you wish to solve 8 optimization problems in parallel. Please note that unlimited academic and commercial licenses are available at **MOSEK**.

5.2 The license system

MOSEK is a commercial product that **always** needs a valid license to work. **MOSEK** uses a third party license manager to implement license checking. The number of license tokens provided determines the number of optimizations that can be run simultaneously.

By default a license token remains checked out from the first optimization until the end of the **MOSEK** session, i.e.

- a license token is checked out when *mosek* is called the first time and

- it is returned when R is terminated, or *mosek_clean* is called.

Starting the optimization when no license tokens are available will result in an error.

Default behaviour of the license system can be changed in several ways:

- Setting the parameter *MSK_IPAR_CACHE_LICENSE* to *"MSK_OFF"* will force **MOSEK** to return the license token immediately after the optimization completed.

```
prob$iparam <- list(CACHE_LICENSE = "MSK_OFF");
```

- Setting the parameter *MSK_IPAR_LICENSE_WAIT* will force **MOSEK** to wait until a license token becomes available instead of returning with an error.

```
prob$iparam <- list(LICENSE_WAIT = "MSK_ON");
```

- The license can be manually released by calling *mosek_clean*.

BASIC TUTORIALS

In this section a number of examples is provided to demonstrate the functionality required for solving linear, conic, semidefinite and quadratic problems as well as mixed integer problems.

- *Basic tutorial* : This is the simplest tutorial: it solves a linear optimization problem read from file. It will show how
  - setup the **MOSEK** environment and problem task,
  - run the solver and
  - check the optimization results.

- *Linear optimization tutorial* : It shows how to input a linear program. It will show how
  - define variables and their bounds,
  - define constraints and their bounds,
  - define a linear objective function,
  - input a linear program but rows or by column.
  - retrieve the solution.

- *Conic quadratic optimization tutorial* : The basic steps needed to formulate a conic quadratic program are introduced:
  - define quadratic cones,
  - assign the relevant variables to their cones.

- *Semidefinite optimization tutorial* : How to input semidefintite optimization problems is the topic of this tutorial, and in particular how to
  - input semidefinite matrices and in sparse format,
  - add semidefinite matrix variable and
  - formulate linear constraints and objective function based on matrix variables.

- *Mixed-Integer optimization tutorial* : This tutorial shows how integrality conditions can be specified.

- *Quadratic optimization tutorial* : It shows how to input quadratic terms in the objective function and constraints.

- *Solution analysis* : This tutorial shows how the user can analyze the solution returned by the solver.

- *Parameter setting tutorial* : This tutorial shows how to set the solver parameters.

6.1 The Basics Tutorial

The simplest program using the **MOSEK** R interface can be described shortly:

1. Load a problem into a problem structure (a *task*).

2. Optimize the problem.

3. Fetch the result.

Listing 6.1: A simple script that reads a problem from file and solves it.

```r
simple <- function( filename)
{

  r <- mosek_read(filename, list(usesol=FALSE, useparam=TRUE))

  if (identical(r$response$code, 0)) {

    print("Successfully read the optimization model")
    prob <- r$prob

    r <- try(mosek(prob, list(verbose=0)), silent=TRUE)
    if (inherits(r, "try-error")) {
      stop("Rmosek failed somehow!")
    }

    if (!identical(r$response$code, 0)) {
      cat(paste("**", "Response code:", r$response$code, "\n"))
      cat(paste("**", r$response$msg, "\n"))
      cat("Trying to continue..\n")
    }

    isdef <- try({
      rbas <- r$sol$bas;
      rbas$solsta;  rbas$prosta;   rbas$xx;
    }, silent=TRUE)
    if (inherits(isdef, "try-error")) {
      stop("Basic solution was incomplete!")
    }

    switch(rbas$solsta,
      OPTIMAL = {
        cat("The solution was optimal, I am happy!\n")
      },
      NEAR_OPTIMAL = {
        cat("The solution was close to optimal, very good..\n")
      },
      #OTHERWISE:
      {
        cat(paste("**", "Solution status:", rbas$solsta, "\n"))
        cat(paste("**", "Problem status:", rbas$prosta, "\n"))
        stop("Solution could not be accepted!")
      }
    )
  }
}
```

6.2 Linear Optimization

The simplest optimization problem is a purely linear problem. A *linear optimization problem* is a problem of the following form:

Minimize or maximize the objective function

$$\sum_{j=0}^{n-1} c_j x_j + c^f$$

subject to the linear constraints

$$l_k^c \leq \sum_{j=0}^{n-1} a_{kj} x_j \leq u_k^c, \quad k = 0, \ldots, m-1,$$

and the bounds

$$l_j^x \leq x_j \leq u_j^x, \quad j = 0, \ldots, n-1.$$

The problem description consists of the following elements:

- m and n — the number of constraints and variables, respectively,
- x — the variable vector of length n,
- c — the coefficient vector of length n

$$c = \begin{bmatrix} c_0 \\ \vdots \\ c_{n-1} \end{bmatrix},$$

- c^f — fixed term in the objective,
- A — an $m \times n$ matrix of coefficients

$$A = \begin{bmatrix} a_{0,0} & \cdots & a_{0,(n-1)} \\ \vdots & \cdots & \vdots \\ a_{(m-1),0} & \cdots & a_{(m-1),(n-1)} \end{bmatrix},$$

- l^c and u^c — the lower and upper bounds on constraints,
- l^x and u^x — the lower and upper bounds on variables.

Please note that we are using 0 as the first index: x_0 is the first element in variable vector x.

6.2.1 Example LO1

The following is an example of a small linear optimization problem:

$$
\begin{array}{rrrrrrrrrl}
\text{maximize} & 3x_0 & + & 1x_1 & + & 5x_2 & + & 1x_3 & & \\
\text{subject to} & 3x_0 & + & 1x_1 & + & 2x_2 & & & = & 30, \\
& 2x_0 & + & 1x_1 & + & 3x_2 & + & 1x_3 & \geq & 15, \\
& & & 2x_1 & & & + & 3x_3 & \leq & 25,
\end{array}
\tag{6.1}
$$

under the bounds

$$
\begin{array}{rclcl}
0 & \leq & x_0 & \leq & \infty, \\
0 & \leq & x_1 & \leq & 10, \\
0 & \leq & x_2 & \leq & \infty, \\
0 & \leq & x_3 & \leq & \infty.
\end{array}
$$

This is easily programmed in R as shown in Listing 6.2. The first line overwrites any previous definitions of the variable *lo1*, preparing for the new problem description. The problem is then defined and finally solved on the last line.

Listing 6.2: R implementation of problem (6.1).

```r
get_lo1_solution_variables <- function(maxtime) {
  lo1 <- list(sense="max", c=c(3,1,5,1))
  lo1$A <- Matrix(c(3,1,2,0,2,1,3,1,0,2,0,3),
                  nrow=3, byrow=TRUE, sparse=TRUE)
  lo1$bc <- rbind(blc=c(30,15,-Inf), buc=c(30,Inf,25));
  lo1$bx <- rbind(blx=c(0,0,0,0),    bux=c(Inf,10,Inf,Inf));
  lo1$dparam <- list(OPTIMIZER_MAX_TIME=maxtime)

  r <- try(mosek(lo1, list(verbose=0)), silent=TRUE)
  if (inherits(r, "try-error")) {
    stop("Rmosek failed somehow!")
  }

  if (!identical(r$response$code, 0)) {
    cat(paste("**", "Response code:", r$response$code, "\n"))
    cat(paste("**", r$response$msg, "\n"))
    cat("Trying to continue..\n")
  }

  isdef <- try({
    rbas <- r$sol$bas;
    rbas$solsta;  rbas$prosta;  rbas$xx;
  }, silent=TRUE)
  if (inherits(isdef, "try-error")) {
    stop("Basic solution was incomplete!")
  }

  switch(rbas$solsta,
    OPTIMAL = {
      cat("The solution was optimal, I am happy!\n")
    },
    NEAR_OPTIMAL = {
      cat("The solution was close to optimal, very good..\n")
    },
    #OTHERWISE:
    {
      cat(paste("**", "Solution status:", rbas$solsta, "\n"))
      cat(paste("**", "Problem status:", rbas$prosta, "\n"))
      stop("Solution could not be accepted!")
    }
  )
  return(rbas$xx)
}
```

Notice how the R value *Inf* is used in both the constraint bounds (*blc* and *buc*) and the variable upper bound (*bux*), to avoid the specification of an actual bound.

From this example the input arguments for the linear program follows easily.

- **Objective** The string is the objective goal and could be either *minimize*, *min*, *maximize* or *max*. The dense numeric vector specifies the coefficients in front of the variables in the linear objective function, and the optional constant scalar (reads: c zero) is a constant in the objective corresponding to c^f in problem , that will be assumed zero if not specified.

- **Constraint Matrix** The sparse matrix is the constraint matrix of the problem with the constraint coefficients written row-wise. Notice that for larger problems it may be more convenient to define an empty sparse matrix and specify the non-zero elements one at a time $A(i, j) = a_{ij}$, rather than writing out the full matrix as done in the *lo1* example. E.g. `Matrix(0,nrow=30,ncol=50,sparse=TRUE)`.

- **Bounds** The constraint bounds with rows *blc* (constraint lower bound) and *buc* (constraint upper

bound), as well as the variable bounds with rows blx (variable lower bound) and bux (variable upper bound), are both given as dense numeric matrices. These are equivalent to the bounds of problem , namely l^c, u^c, l^x and u^x.

6.3 Conic Quadratic Optimization

Conic optimization is a generalization of linear optimization, allowing constraints of the type

$$x^t \in \mathcal{K}_t,$$

where x^t is a subset of the problem variables and $\mathcal{K}_t$ is a convex cone. Since the set $\mathbb{R}^n$ of real numbers is also a convex cone, we can simply write a compound conic constraint $x \in \mathcal{K}$ where $\mathcal{K} = \mathcal{K}_1 \times \cdot \times \mathcal{K}_l$ is a product of smaller cones and x is the full problem variable.

MOSEK can solve conic quadratic optimization problems of the form

$$
\begin{array}{llrcl}
\text{minimize} & & c^T x + c^f & & \\
\text{subject to} & l^c & \leq \quad Ax & \leq & u^c, \\
& l^x & \leq \quad x & \leq & u^x, \\
& & x \in \mathcal{K}, & &
\end{array}
$$

where the domain restriction, $x \in \mathcal{K}$, implies that all variables are partitioned into convex cones

$$x = (x^0, x^1, \ldots, x^{p-1}), \quad \text{with } x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}.$$

For convenience, a user defining a conic quadratic problem only needs to specify subsets of variables x^t belonging to quadratic cones. These are:

- Quadratic cone:

$$\mathcal{Q}^n = \left\{ x \in \mathbb{R}^n : x_0 \geq \sqrt{\sum_{j=1}^{n-1} x_j^2} \right\}.$$

- Rotated quadratic cone:

$$\mathcal{Q}_r^n = \left\{ x \in \mathbb{R}^n : 2x_0 x_1 \geq \sum_{j=2}^{n-1} x_j^2, \quad x_0 \geq 0, \quad x_1 \geq 0 \right\}.$$

For example, the following constraint:

$$(x_4, x_0, x_2) \in \mathcal{Q}^3$$

describes a convex cone in $\mathbb{R}^3$ given by the inequality:

$$x_4 \geq \sqrt{x_0^2 + x_2^2}.$$

Furthermore, each variable may belong to one cone at most. The constraint $x_i - x_j = 0$ would however allow x_i and x_j to belong to different cones with same effect.

6.3.1 Example CQO1

Consider the following conic quadratic problem which involves some linear constraints, a quadratic cone and a rotated quadratic cone.

$$
\begin{array}{lrcl}
\text{minimize} & x_4 + x_5 + x_6 & & \\
\text{subject to} & x_1 + x_2 + 2x_3 & = & 1, \\
& x_1, x_2, x_3 & \geq & 0, \\
& x_4 \geq \sqrt{x_1^2 + x_2^2}, & & \\
& 2x_5 x_6 \geq x_3^2 & &
\end{array}
\tag{6.2}
$$

The first cone is of the quadratic cone type (`"MSK_CT_QUAD"`), while the second is of the rotated quadratic cone type (`"MSK_CT_RQUAD"` .) The subindexes of the variables used to define these cones follow naturally as seen in the Listing 6.3.

Listing 6.3: R implementation of model (6.2)

```
cqo1 <- list(sense = "min")
cqo1$c <- c(0,0,0,1,1,1)
cqo1$A <- Matrix(c(1,1,2,0,0,0),
                 nrow=1, byrow=TRUE, sparse=TRUE)
cqo1$bc <- rbind(blc = 1, buc = 1)
cqo1$bx <- rbind(blx = c(0,0,0,-Inf,-Inf,-Inf),
                 bux = rep(Inf,6))
cqo1$cones <- cbind(
  list("QUAD", c(4,1,2)),
  list("RQUAD", c(5,6,3))
)
rownames(cqo1$cones) <- c("type","sub");
r <- mosek(cqo1)
```

From this example the input arguments for a conic program follow easily. The objective function, the linear constraints and variable bounds should all be specified as for linear programs, and the only addition to this is the quadratic cones specified in the list-typed matrix.

The *cones* matrix has a column for each cone, and a row for each descriptive element. The first row called , should specify the cone type in a string, being either quadratic `QUAD` or rotated quadratic `RQUAD`. Notice that the **MOSEK** library cone type prefix `MSK_CT_` is optional. The second row called , should specify the subset of variables belonging to the cone in a numeric vector - and the ordering does matter! The i'th element of *sub* will be the index of the variable referred by x_i, in the cone definitions and . As an example, the rotated quadratic cone with subindexes `c(4,6,2,3)` would define the cone

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^4 \ : \ 2x_4 x_6 \geq x_2^2 + x_3^2, \ x_4 \geq 0, \ x_6 \geq 0 \right\}.$$

Listing 6.4 showed a simple way to specify cones given an explicit representation. In many practical cases, however, cones are more conveniently specified in chunks or within a loop. For this purpose, preallocation should always be preferred as shown here.

Listing 6.4: R implementation of model (6.2) preallocating cones.

```
NUMCONES <- 2
cqo1$cones <- matrix(list(), nrow=2, ncol=NUMCONES)
  rownames(cqo1$cones) <- c("type","sub")
cqo1$cones[,1] <- list("QUAD", c(5,1,3))
cqo1$cones[,2] <- list("QUAD", c(6,2,4))
```

6.4 Semidefinite Optimization

Semidefinite optimization is a generalization of conic quadratic optimization, allowing the use of matrix variables belonging to the convex cone of positive semidefinite matrices

$$\mathcal{S}_+^r = \left\{ X \in \mathcal{S}^r : z^T X z \geq 0, \quad \forall z \in \mathbb{R}^r \right\},$$

where $\mathcal{S}^r$ is the set of $r \times r$ real-valued symmetric matrices.

MOSEK can solve semidefinite optimization problems of the form

$$
\begin{aligned}
\text{minimize} \quad & \sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f \\
\text{subject to} \quad l_i^c \leq \ & \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle && \leq \ u_i^c, \quad i = 0, \ldots, m-1, \\
l_j^x \leq \ & \qquad\qquad x_j && \leq \ u_j^x, \quad j = 0, \ldots, n-1, \\
& x \in \mathcal{K}, \overline{X}_j \in \mathcal{S}_+^{r_j}, && \qquad\qquad j = 0, \ldots, p-1
\end{aligned}
$$

where the problem has p symmetric positive semidefinite variables $\overline{X}_j \in \mathcal{S}_+^{r_j}$ of dimension r_j with symmetric coefficient matrices $\overline{C}_j \in \mathcal{S}^{r_j}$ and $\overline{A}_{i,j} \in \mathcal{S}^{r_j}$. We use standard notation for the matrix inner product, i.e., for $A, B \in \mathbb{R}^{m \times n}$ we have

$$\langle A, B \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} A_{ij} B_{ij}.$$

6.4.1 Example SDO1

We consider the simple optimization problem with semidefinite and conic quadratic constraints:

$$
\begin{aligned}
\text{minimize} \quad & \left\langle \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \overline{X} \right\rangle + x_0 \\
\text{subject to} \quad & \left\langle \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \overline{X} \right\rangle + x_0 & = & 1, \\
& \left\langle \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \overline{X} \right\rangle + x_1 + x_2 & = & 1/2, \\
& x_0 \geq \sqrt{{x_1}^2 + {x_2}^2}, & \overline{X} \succeq 0,
\end{aligned}
\tag{6.3}
$$

The problem description contains a 3-dimensional symmetric semidefinite variable which can be written explicitly as:

$$
\overline{X} = \begin{bmatrix} \overline{X}_{00} & \overline{X}_{10} & \overline{X}_{20} \\ \overline{X}_{10} & \overline{X}_{11} & \overline{X}_{21} \\ \overline{X}_{20} & \overline{X}_{21} & \overline{X}_{22} \end{bmatrix} \in \mathcal{S}_+^3,
$$

and a conic quadratic variable $(x_0, x_1, x_2) \in \mathcal{Q}^3$. The objective is to minimize

$$2(\overline{X}_{00} + \overline{X}_{10} + \overline{X}_{11} + \overline{X}_{21} + \overline{X}_{22}) + x_0,$$

subject to the two linear constraints

$$
\begin{aligned}
\overline{X}_{00} + \overline{X}_{11} + \overline{X}_{22} + x_0 & = & 1, \\
\overline{X}_{00} + \overline{X}_{11} + \overline{X}_{22} + 2(\overline{X}_{10} + \overline{X}_{20} + \overline{X}_{21}) + x_1 + x_2 & = & 1/2.
\end{aligned}
$$

and can be modeled in R as shown in Listing 6.5.

Listing 6.5: R implementation of model (6.3).

```r
sdo1 <- list(sense="min")
sdo1$c      <- c(1,0,0)
sdo1$A      <- Matrix(c(1,0,0,
                        0,1,1), nrow=2, byrow=TRUE, sparse=TRUE)
sdo1$bc     <- rBind(blc = c(1, 0.5), buc = c(1, 0.5))
sdo1$bx     <- rBind(blx = rep(-Inf,3), bux = rep(Inf,3))
sdo1$cones <- cBind(list("quad", c(1,2,3)))

# One semidefinite matrix variable size 3x3:
N <- 3
sdo1$bardim <- c(N)

# Block triplet format specifying the lower triangular part
# of the symmetric coefficient matrix 'barc':
sdo1$barc$j <- c(1, 1, 1, 1, 1)
sdo1$barc$k <- c(1, 2, 3, 2, 3)
sdo1$barc$l <- c(1, 2, 3, 1, 2)
sdo1$barc$v <- c(2, 2, 2, 1, 1)
```

```
# Block triplet format specifying the lower triangular part
# of the symmetric coefficient matrix 'barA':
sdo1$barA$i <- c(1, 1, 1, 2, 2, 2, 2, 2, 2)
sdo1$barA$j <- c(1, 1, 1, 1, 1, 1, 1, 1, 1)
sdo1$barA$k <- c(1, 2, 3, 1, 2, 3, 2, 3, 3)
sdo1$barA$l <- c(1, 2, 3, 1, 2, 3, 1, 1, 2)
sdo1$barA$v <- c(1, 1, 1, 1, 1, 1, 1, 1, 1)

r <- mosek(sdo1)
barx    <- 1.0 * bandSparse(N, k=0:(1-N), symm=TRUE)
barx@x <- r$sol$itr$barx[[1]]
```

6.5 Quadratic Optimization

MOSEK can solve quadratic and quadratically constrained convex problems. This class of problems can be formulated as follows:

$$
\begin{array}{lllll}
\text{minimize} & & \frac{1}{2}x^T Q^o x + c^T x + c^f & & \\
\text{subject to} & l_k^c \leq & \frac{1}{2}x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j & \leq u_k^c, & k = 0, \ldots, m-1, \\
& l_j^x \leq & x_j & \leq u_j^x, & j = 0, \ldots, n-1.
\end{array}
\tag{6.4}
$$

Without loss of generality it is assumed that Q^o and Q^k are all symmetric because

$$
x^T Q x = \frac{1}{2}x^T(Q + Q^T)x.
$$

This implies that a non-symmetric Q can be replaced by the symmetric matrix $\frac{1}{2}(Q + Q^T)$.

The problem is required to be convex. More precisely, the matrix Q^o must be positive semi-definite and the kth constraint must be of the form

$$
l_k^c \leq \frac{1}{2}x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j
\tag{6.5}
$$

with a negative semi-definite Q^k or of the form

$$
\frac{1}{2}x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \leq u_k^c.
$$

with a positive semi-definite Q^k. This implies that quadratic equalities are *not* allowed. Specifying a non-convex problem will result in an error when the optimizer is called.

A matrix is positive semidefinite if the smallest eigenvalue of the matrix is nonnegative. An alternative statement of the positive semidefinite requirement is

$$
x^T Q x \geq 0, \quad \forall x.
$$

If Q is not positive semidefinite, then **MOSEK** will not produce reliable results or work at all.

One way of checking whether Q is positive semidefinite is to check whether all the eigenvalues of Q are nonnegative.

6.5.1 Example: Quadratic Objective

$$
\begin{array}{lll}
\text{minimize} & & x_1^2 + 0.1x_2^2 + x_3^2 - x_1 x_3 - x_2 \\
\text{subject to} & 1 \leq & x_1 + x_2 + x_3 \\
& & x \geq 0.
\end{array}
\tag{6.6}
$$

For the example (6.6) implies that

$$Q = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0.2 & 0 \\ -1 & 0 & 2 \end{bmatrix}, c = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix},$$

and that

$$l^c = 1, u^c = \infty, l^x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } u^x = \begin{bmatrix} \infty \\ \infty \\ \infty \end{bmatrix}$$

Please note the explicit $\frac{1}{2}$ in the objective function of (6.4) which implies that diagonal elements must be doubled in Q, i.e. $Q_{11} = 2$, whereas the coefficient in (6.6) is 1 in front of x_1^2.

Important: **MOSEK** assumes that the Q matrix is symmetric, i.e. $Q = Q^T$, and that Q is *positive semidefinite*.

Please note that there are quadratic terms in both constraints. This problem can be solved using *mosek* as reported in Listing 6.6.

Listing 6.6: Script implementing problem (6.6)

```
qo1 <- list()
qo1$sense <- "min"
qo1$c <- c(0,-1,0)
qo1$A <- Matrix(c(1,1,1), nrow=1, byrow=TRUE, sparse=TRUE)
qo1$bc <- rbind(blc = 1,
                buc = Inf)
qo1$bx <- rbind(blx = rep(0,3),
                bux = rep(Inf,3))

qo1$qobj <- list(i = c(1,  3,   2, 3),
                 j = c(1,  1,   2, 3),
                 v = c(2, -1, 0.2, 2))

r <- mosek(qo1)
```

6.6 Integer Optimization

An optimization problem where one or more of the variables are constrained to integer values is called a (mixed) integer optimization problem. **MOSEK** supports integer variables in combination with linear and conic quadratic problems. See the previous tutorials for an introduction to how to model these types of problems.

6.6.1 Example MILO1

We use the example

$$\begin{array}{lrcl} \text{maximize} & x_0 + 0.64x_1 \\ \text{subject to} & 50x_0 + 31x_1 & \leq & 250, \\ & 3x_0 - 2x_1 & \geq & -4, \\ & x_0, x_1 \geq 0 & & \text{and integer} \end{array} \qquad (6.7)$$

to demonstrate how to set up and solve a problem with integer variables. It has the structure of a linear optimization problem (see *6.2*) except for integrality constraints on the variables. Therefore, only the specification of the integer constraints requires something new compared to the linear optimization problem discussed previously.

This is easily programmed in R using the piece code shown in Listing 6.7,

Listing 6.7: R implementation of problem (6.7).

```
milo1 <- list(sense = "max")
milo1$c <- c(1, 0.64)
milo1$A <- Matrix(c( 50, 31 ,
                      3, -2 ),
               nrow  = 2,
               byrow = TRUE,
               sparse= TRUE)
milo1$bc <-
  rbind(blc = c(-Inf,-4),
        buc = c(250,Inf))
milo1$bx <-
  rbind(blx = c(0,0),
        bux = c(Inf,Inf))
milo1$intsub <- c(1,2)
r <- mosek(milo1)
```

where x_1 and x_2 are pointed out as integer variables.

The input arguments follow those of a linear or conic program with the additional identification of the integer variables. The column vector `intsub` should simply contain indexes to the subset of variables for which integrality is required. For instance if x should be a binary $\{0,1\}$-variable, its index in the problem formulation should be added to `intsub`, and its bounds $0 \le x \le 1$ should be specified explicitly.

If executed correctly you should be able to see the log of the interface and optimization process printed to the screen. The output structure will only include an integer solution `int`, since we are no longer in the continuous domain for which the interior-point algorithm operates. The structure also contains the problem status as well as the solution status based on certificates found by the **MOSEK** optimization library.

Solving a mixed-integer optimization program could easily result in long running time. It is therefore of interest to consider a termination criterion based on the maximum running time. This is possible setting the *MSK_DPAR_MIO_MAX_TIME*. See Section *6.8* for more details on how to set solver parameters.

6.6.2 Specifying an initial solution

Solution time of can often be reduced by providing an initial solution for the solver. It is not necessary to specify the whole solution. By setting the *MSK_IPAR_MIO_CONSTRUCT_SOL* parameter to *"MSK_ON"* and inputting values for the integer variables only, **MOSEK** will be forced to compute the remaining continuous variable values. If the specified integer solution is infeasible or incomplete, **MOSEK** will simply ignore it.

We concentrate on a simple example below.

$$
\begin{array}{lll}
\text{maximize} & 7x_0 + 10x_1 + x_2 + 5x_3 \\
\text{subject to} & x_0 + x_1 + x_2 + x_3 \le 2.5 \\
& x_0, x_1, x_2 \in \mathbb{Z} \\
x_0, x_1, x_2, x_3 \ge 0 &
\end{array}
\tag{6.8}
$$

The following example demonstrates how to optimize the problem using a feasible starting solution generated by selecting the integer values as $x_0 = 0, x_1 = 2, x_2 = 0$.

6.7 Solution Analysis

The main purpose of **MOSEK** is to solve optimization problems and therefore the most fundamental question to be asked is whether the solution reported by **MOSEK** is a solution to the desired optimization problem.

There can be several reasons why it might be not case. The most prominent reasons are:

- A wrong problem. The problem inputted to **MOSEK** is simply not the right problem, i.e. some of the data may have been corrupted or the model has been incorrectly built.

- Numerical issues. The problem is badly scaled or otherwise badly posed.

- Other reasons. E.g. not enough memory or an explicit user request to stop.

The first step in verifying that **MOSEK** reports the expected solution is to inspect the solution summary generated by **MOSEK** (see Section *6.7.1*). The solution summary provides information about

- the problem and solution statuses,

- objective value and infeasibility measures for the primal solution, and

- objective value and infeasibility measures for the dual solution, where applicable.

If the summary reports conflicting information (e.g. a solution status that does not match the actual solution), or the cause for terminating the solver before a solution was found cannot be traced back to the reasons stated above, it may be caused by a bug in the solver; in this case, please contact **MOSEK** support (see Section *2*).

If it has been verified that **MOSEK** solves the problem correctly but the solution is still not as expected, next step is to verify that the primal solution satisfies all the constraints. Hence, using the original problem it must be determined whether the solution satisfies all the required constraints in the model. For instance assume that the problem has the constraints

$$\begin{aligned} x_1 + 2x_2 + x_3 &\leq 1, \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

and **MOSEK** reports the optimal solution

$$x_1 = x_2 = x_3 = 1.$$

Then clearly the solution violates the constraints. The most likely explanation is that the model does not match the problem entered into **MOSEK**, for instance

$$x_1 - 2x_2 + x_3 \leq 1$$

may have been inputted instead of

$$x_1 + 2x_2 + x_3 \leq 1.$$

A good way to debug such an issue is to dump the problem to *OPF file* and check whether the violated constraint has been specified correctly.

Verifying that a feasible solution is optimal can be harder. However, for continuous problems, i.e. problems without any integer constraints, optimality can verified using a dual solution. Normally, **MOSEK** will report a dual solution; if that is feasible and has the same objective value as the primal solution, then the primal solution must be optimal.

An alternative method is to find another primal solution that has better objective value than the one reported to **MOSEK**. If that is possible then either the problem is badly posed or there is bug in **MOSEK**.

6.7.1 The Solution Summary

Due to **MOSEK** employs finite precision floating point numbers then reported solution is an approximate optimal solution. Therefore after solving an optimization problem it is relevant to investigate how good an approximation the solution is. For a convex optimization problem that is an easy task because the optimality conditions are:

- The primal solution must satisfy all the primal constraints.

- The dual solution much satisfy all the dual constraints.

- The primal and dual objective values must be identical.

Therefore, the **MOSEK** solution summary displays that information that makes it possible to verify the optimality conditions. Indeed the solution summary reports how much primal and dual solutions violate the primal and constraints respectively. In addition the objective values assoctaied with each solution repoted.

In case of a linear optimization problem the solution summary may look like

```
Basic solution summary
  Problem status  : PRIMAL_AND_DUAL_FEASIBLE
  Solution status : OPTIMAL
  Primal.  obj: -4.6475314286e+002  nrm: 5e+002   Viol.  con: 1e-014   var: 1e-014
  Dual.    obj: -4.6475314543e+002  nrm: 1e+001   Viol.  con: 4e-009   var: 4e-016
```

The interpreation of the solution summary is as follows:

- Information for the basic solution is reported.

- The problem status is primal and dual feasible which means the problem has an optimal solution.

- The solution status is optimal.

- Next information about the primal solution is reported. The information consists of the objective value, the infinity norm of the primal solution and violation meassures. The violation for the constraints (`con:`) is the maximal violation in any of the constraints. Whereas the violations for the variables (`var:`) is the maximal bound violation for any of the variables. In this case the primal violations for the constraints and variables are small meaning the solution is an almost feasible solution. Observe due to the rounding errors it can be expected that the violations are proportional to the size (`nrm:`) of the solution.

- Similarly for the dual solution the violations are small and hence the dual solution is almost feasible.

- Finally, it can be seen that the primal and dual objective values are almost identical.

To summarize in this case a primal and a dual solution only violate the primal and dual constraints slightly. Moreover, the primal and dual objective values are almost identical and hence it can be concluded that the reported solution is a good approximation to the optimal solution.

The reason the size (=norms) of the solution are shown is that it shows some about conditioning of the problem because if the primal and/or dual solution has very large norm then the violations and objective values are sensitive to small pertubations in the problem data. Therefore, the problem is unstable and care should be taken before using the solution.

Now what happens if the problem does not have an optimal solution e.g. is primal infeasible. In such a case the solution summary may look like

```
Interior-point solution summary
  Problem status  : PRIMAL_INFEASIBLE
  Solution status : PRIMAL_INFEASIBLE_CER
  Dual.    obj: 6.7319732555e+000   nrm: 8e+000   Viol.  con: 3e-010   var: 2e-009
```

i.e. **MOSEK** reports that the solution is a certificate of primal infeasibility but a certificate of primal infeasibility what does that mean? It means that the dual solution is a Farkas type certificate. Recall Farkas' Lemma says

$$
\begin{aligned}
Ax &= b, \\
x &\geq 0
\end{aligned}
$$

if and only if a y exists such that

$$
\begin{aligned}
A^T y &\leq 0, \\
b^T y &> 0.
\end{aligned}
\tag{6.9}
$$

Observe the infeasibility certificate has the same form as a regular dual solution and therefore the certificate is stored as a dual solution. In order to check quality of the primal infeasibility certificate

it should be checked whether satisfies (6.9). Hence, the dual objective value is $b^T y$ should be strictly positive and the maximal violation in $A^T y \leq 0$ should be a small. In this case we conclude the certificate is of high quality because the dual objective is postive and large compared to the violations. Note the Farkas certificate is a ray so any postive multiple of that ray is also certificate. This implies the absolute of the value objective value and the violation is not relevant.

In the case a problem is dual infeasible then the solution summary may look like

```
Basic solution summary
Problem status  : DUAL_INFEASIBLE
Solution status : DUAL_INFEASIBLE_CER
Primal.  obj: -2.0000000000e-002  nrm: 1e+000   Viol.  con: 0e+000   var: 0e+000
```

Observe when a solution is a certificate of dual infeasibility then the primal solution contains the certificate. Moreoever, given the problem is a minimization problem the objective value should be negative and large compared to the worst violation if the certificate is strong.

Listing 6.8 shows how to use these function to determine the quality of the solution.

Listing 6.8: An example of solution quality analysis.

```r
lo1 <- list(sense="max", c=c(3,1,5,1))
lo1$A <- Matrix(c(3,1,2,0,2,1,3,1,0,2,0,3),
          nrow=3, byrow=TRUE, sparse=TRUE)
lo1$bc <- rbind(blc=c(30,15,-Inf), buc=c(30,Inf,25));
lo1$bx <- rbind(blx=c(0,0,0,0),    bux=c(Inf,10,Inf,Inf));

## Note we need to obtain details of the solution to inspect primal/dual obj fun value
r <- try(mosek(lo1, list(verbose=0, soldetail=2)), silent=TRUE)

if (inherits(r, "try-error")) {
  stop("Rmosek failed somehow!")
}

if (!identical(r$response$code, 0)) {
  cat(paste("**", "Response code:", r$response$code, "\n"))
  cat(paste("**", r$response$msg, "\n"))
  cat("Trying to continue..\n")
}

isdef <- try({
  itr <- r$sol$itr;
  itr$solsta;  itr$prosta;  itr$xx; itr$sol;
  }, silent=TRUE)

if (inherits(isdef, "try-error")) {
    stop("Interior solution was incomplete!")
}

if (itr$solsta == "OPTIMAL" |
    itr$solsta == "NEAR_OPTIMAL" ) {

  cat("The solution was (near) optimal, I am happy!\n")

  pobj<- r$sol$itr$pobjval
  dobj<- r$sol$itr$dobjval

  abs_obj_gap     <- abs(dobj-pobj)
  rel_obj_gap     <- abs_obj_gap/(1.0 + min(abs(pobj),abs(dobj)))


  max_primal_viol <- max( itr$maxinfeas$pcon, itr$maxinfeas$pbound)
  max_primal_viol <- max( max_primal_viol  , itr$maxinfeas$pbarvar)
  max_primal_viol <- max( max_primal_viol  , itr$maxinfeas$pcone)
```

```r
  max_dual_viol <- max( itr$maxinfeas$dcon, itr$maxinfeas$dbound)
  max_dual_viol <- max( max_dual_viol    , itr$maxinfeas$dbarvar)
  max_dual_viol <- max( max_dual_viol    , itr$maxinfeas$dcone)


  ## Assume the application needs the solution to be within
  ## 1e-6 of optimality in an absolute sense. Another approach
  ## would be looking at the relative objective gap

  cat("\n\n")
  cat("Customized solution information.\n")
  cat("  Absolute objective gap: ",abs_obj_gap,"\n")
  cat("  Relative objective gap: ",rel_obj_gap,"\n")
  cat("  Max primal violation  : ",max_primal_viol,"\n")
  cat("  Max dual violation    : ",max_dual_viol,"\n")

  accepted<- TRUE

  if (rel_obj_gap>1e-6){
    print ("Warning: The relative objective gap is LARGE.")
    accepted <- FALSE
  }

  ## We will accept a primal infeasibility of 1e-8 and
  ## dual infeasibility of 1e-6. These number should chosen problem
  ## dependent.

  if ( max_primal_viol>1e-8) {
    print ("Warning: Primal violation is too LARGE")
    accepted <- FALSE
  }

  if ( max_dual_viol>1e-6 ) {
    print ("Warning: Dual violation is too LARGE.")
    accepted <- FALSE
  }

  if ( accepted ) {
    numvar <- task.getnumvar()
    print ("Optimal primal solution")
    print(rbas$xx)
  }

} else {
    #Print detailed information about the solution
    cat(paste("**", "Solution status:", rbas$solsta, "\n"))
    cat(paste("**", "Problem status:", rbas$prosta, "\n"))
    stop("Solution could not be accepted!")
}
```

6.7.2 The Solution Summary for Mixed-Integer Problems

The solution summary for a mixed-integer problem may look like

Listing 6.9: Example of solution summary for a mixed-integer problem.

```
Integer solution solution summary
Problem status  : PRIMAL_FEASIBLE
Solution status : INTEGER_OPTIMAL
Primal.  obj: 3.4016000000e+005   nrm: 1e+000   Viol.  con: 0e+000   var: 0e+000   itg: 3e-014
```

The main diffrence compared to thecontinous case covered previously is that no information about the dual solution is provided. Simply because there is no dual solution available for a mixed integer problem. In this case it can be seen that the solution is highly feasible because the violations are small. Moreoever, the solution is denoted integer optimal. Observe *itg: 3e-014* implies that all the integer constrained variables are at most $3e - 014$ from being an exact integer.

6.8 Solver Parameters

The **MOSEK** API provides many parameters to tune and customize the solver behaviour. Parameters are grouped depending on their type: integer, double or string. In general, it should not be necessary to change any of the parameters but if required, it is easily done. A complete list of all parameters is found in Section *12.3*.

We will show how to access and set the integer parameter that define the logging verbosity of the solver, i.e. *MSK_IPAR_LOG*, and the algorithm used by **MOSEK**, i.e. *MSK_IPAR_OPTIMIZER*.

Note: The very same concepts and procedures apply to string and double valued parameters.

We need to set to *0* the parameter *MSK_IPAR_LOG*. Notice that in Rmosek the MSK_IPAR_, MSK_DPAR_ and MSK_SPAR_ prefixes for parameter names have been removed in favor of the `iparam`, `dparam` and `sparam` structures. Parameters are case-insensitive.

Assuming a problem named *lo1* has been defined, we can suppress solver verbosity as follow

```
lo1$iparam <- list(LOG = 0);
```

For more information about other parameter related functions, please browse the API reference in Section *12*.

Setting a parameter to `NULL` will remove it from the list according to the R language definition. Setting a parameter to `NA` or `NaN`, will on the other hand keep it on the list, only to be ignored by the interface with warnings confirming that this took place. Errors will be generated when a parameter name is not recognized or when the value defined for it is not within its feasible range.

NONLINEAR TUTORIALS

This chapter provides information about how to solve general convex nonlinear optimization problems using **MOSEK**. By general nonlinear problems it is meant problems that cannot be formulated as a conic quadratic optimization or a convex quadratically constrained optimization problem.

In general it is recommended not to use nonlinear optimizer unless needed. The reasons are

- **MOSEK** has no way of checking whether the formulated problem is convex and if this assumption is not satisfied the optimizer will not work.

- The nonlinear optimizer requires 1st and 2nd order derivative information which is hard to provide correctly i.e. it is nontrivial to program the code that computes the derivative information.

- The algorithm employed for nonlinear optimization problems is not as good as the one employed for conic problems i.e. conic problems has special that can be exploited to make the optimizer faster and more robust.

This leads to following advices in decreasing order of importance.

1. Consider reformulating the problem to a conic quadratic optimization problem if at all possible. In particular many problems involving polynomial terms can easily be reformulated to conic quadratic form.

2. Consider reformulating the problem to a separable optimization problem because that simplifies the issue with verifying convexity and computing 1st and 2nd order derivatives significantly. In most cases problems on separable form also solves faster because of the simpler structure of the functions.

3. Finally, if the problem cannot be reformulated to separable form then use a modelling language like AMPL or GAMS. The reason is the modeling language will do all the computing of function values and derivatives. This eliminates an important source of errors. Therefore, it is strongly recommended to use a modelling language at the prototype stage.

7.1 Separable Convex (SCopt) Interface

The **MOSEK** rmosek API provides a way to add simple non-linear functions composed from a limited set of non-linear terms. Non-linear terms can be mixed with quadratic terms in objective and constraints. We consider a normal linear problem with additional non-linear terms z:

$$
\begin{aligned}
\text{minimize} \quad & z_0(x) + c^T x \\
\text{subject to} \quad l_i^c \leq \quad & z_i(x) + a_i^T x \leq u_i^c, \ i = 1 \ldots m \\
l^x \leq \quad & x \leq u^x, \\
x \in \mathbb{R}^n \quad z : \mathbb{R}^n & \to \mathbb{R}^{(m+1)}
\end{aligned}
$$

With the separable convex interface SCopt it is possible to add non-linear functions of the form

$$
z_i(x) = \sum_{k=1}^{K_i} w_k^i(x_{p_{ik}}), \ w_k^i : \mathbb{R} \to \mathbb{R}
$$

In other words, each non-linear function z_i is a sum of separable functions w_k^i of one variable each. A limited set of functions is supported; each w_k^i can be one of the following:

Table 7.1: Functions supported by the SCopt interface.

Separable function	Operator name	Name
$f x \ln(x)$	ent	Entropy function
$f e^{gx+h}$	exp	Exponential function
$f \ln(gx + h)$	log	Logarithm
$f(x + h)^g$	pow	Power function

where f, g and h are constants. This formulation does not guarantee convexity. For **MOSEK** to be able to solve the problem, following requirements must be met:

- If the objective is minimized, the sum of non-linear terms must be convex, otherwise it must be concave.

- Any constraint bounded below must be concave, and any constraint bounded above must be convex.

- Each separable term must be twice differentiable within the bounds of the variable it is applied to.

Some simple rules can be followed to ensure that the problem satisfies **MOSEK**'s convexity and differentiability requirements. First of all, for any variable x_i used in a separable term, the variable bounds must define a range within which the function is twice differentiable.

We can define these bounds as follows:

Separable function	Operator name	Safe x bounds
$f x \ln(x)$	ent	$0 < x$.
$f e^{gx+h}$	exp	$-\infty < x < \infty$.
$f \ln(gx + h)$	log	If $g > 0$: $-h/g < x$.
		If $g < 0$: $x < -h/g$.
$f(x + h)^g$	pow	If $g > 0$ and integer: $-\infty < x < \infty$.
		If $g < 0$ and integer: either $-h < x$ or $x < -h$.
		Otherwise: $-h < x$.

To ensure convexity, we require that each $z_i(x)$ is either a sum of convex terms or a sum of concave terms. The following table lists convexity conditions for the relevant ranges for $f > 0$ — changing the sign of f switches concavity/convexity.

Separable function	Operator name	Convexity conditions
$f x \ln(x)$	ent	Convex within safe bounds.
$f e^{gx+h}$	exp	Convex for all x.
$f \ln(gx + h)$	log	Concave within safe bounds.
$f(x + h)^g$	pow	If g is even integer: convex within safe bounds.
		If g is odd integer: • concave if $(-\infty, -h)$, • convex if $(-h, \infty)$ If $0 < g < 1$: concave within safe bounds.
		Otherwise: convex within safe bounds.

Important: Do NOT use the SCopt API unless you really have to. First try to reformulate the problem in conic form. The SCopt API is less stable, less predictable, harder to debug and has inferior status info.

A problem involving linear combinations of variables (for example under the logarithm function), can be converted to a separable problem using slack variables and additional constraints. The evaluation of the nonlinear functions, their gradients and Hessians is much easier in the separable case.

7.1.1 Example

Consider the following separable convex problem:

$$
\begin{array}{ll}
\text{minimize} & \exp(x_2) - \ln(x_1) \\
\text{subject to} & x_2 \ln(x_2) \leq 0 \\
& x_1^{1/2} - x_2 \geq 0 \\
& \tfrac{1}{2} \leq x_1, x_2 \leq 1.
\end{array}
\tag{7.1}
$$

Note that all nonlinear functions are well defined for x values satisfying the variable bounds strictly. This assures that function evaluation errors will not occur during the optimization process because **MOSEK**.

From this example the input arguments for a separable convex program follow easily. The linear part of the objective function and constraints, as well as the constraint and variable bounds, should all be specified as for linear programs (see Section *6.2*.). The only addition to this is the list called *opro* containing the list-typed operator matrices (for objective) and (for constraints).

```
##                type  j      f     g     h
opro[,1] <- list("LOG", 1,  -1.0, 1.0, 0.0)
opro[,2] <- list("EXP", 2,   1.0, 1.0, 0.0)
```

The operator matrices have a column for each operator and a row for each descriptive element. The `opro` matrix have five rows called $\{type, j, f, g, h\}$, while the `oprc` matrix have six rows called $\{type, i, j, f, g, h\}$. Row should specify the operator type in a string, being either, exponential *EXP*, logarithm *LOG* or power *POW*. Row (not in *opro*) should specify the index of the constraint to which the non-linear term should be added. Row should specify the variable index of the operator. Rows , and should specify the coefficients of the operator.

```
##                type  i      j    f     g     h
oprc[,1] <- list("ENT", 1,   2, 1.0, 0.0, 0.0)
oprc[,2] <- list("POW", 2,   1, 1.0, 0.5, 0.0)
```

Note that the definition of the entropy operator, was the only operator defined without g and h. Thus, for entropy operators, these two unused rows in the operator matrix can be set to either zero or any empty definition (`NULL`, `NA` or `NaN`).

MANAGING I/O

The main purpose of this chapter is to give an overview on the logging and I/O features provided by the **MOSEK** package.

- Section *8.1* contains information about the log streams provided by **MOSEK**.

- File I/O is discussed in Section *8.2*.

- How to tune the logging verbosity is the topic of Section *8.3*.

8.1 Stream I/O

MOSEK execution produces a certain amount of loggging at environment and task level. This means that the logging from each environement and task can be isolated from the others.

The log messages are partitioned in three streams:

- *messages*

- *warnings*

- *errors*

These streams are aggregated in the *log* stream.

8.2 File I/O

MOSEK supports a range of problem and solution formats listed in Section *13*. One such format is **MOSEK**'s native binary *Task format* which supports all features that **MOSEK** supports.

The file format used in I/O operations is deduced from extension - as in `problemname.task` - unless the parameter *MSK_IPAR_WRITE_DATA_FORMAT* is specified to something else. Problem files with an additional `.gz` extension - as in `problemname.task.gz` - are moreover assumed to use GZIP compression, and are automatically compressed, respectively decompressed, when written or read.

Example

If something is wrong with a problem or a solution, one option is to output the problem to the human-readable *OPF format* and inspect it by hand. For instance, one may use the *mosek_write* function to write the problem to a file immediately before optimizing it:

```
r <- mosek_write(lo1, "lo1.opf")
```

This will write the problem in `lo1` to the file `lo1.opf`. Similarly, using *mosek_read* is possible to read a problem from file. For instance the following code line

```
r <- mosek_read("lo1.opf")
```

will import a problem stored in the file `lo1.opf` in an object called *prob*.

8.3 Verbosity

The logging verbosity can be controlled by setting the relevant paramenters, as for instance

- *MSK_IPAR_LOG* ,
- *MSK_IPAR_LOG_INTPNT* ,
- *MSK_IPAR_LOG_MIO* ,
- *MSK_IPAR_LOG_CUT_SECOND_OPT* ,
- *MSK_IPAR_LOG_SIM* , and
- *MSK_IPAR_LOG_SIM_MINOR* .

Each parameter control the output level of a specific functionality or algorithm. The main switch is *MSK_IPAR_LOG* which affect the whole output. The actual log level for a specific functionality is determined as the minimum between *MSK_IPAR_LOG* and the relevant parameter. For instance, the log level for the output produce by the interior-point algorithm is tuned by the *MSK_IPAR_LOG_INTPNT*: the actual log level is defined by the minimum between *MSK_IPAR_LOG* and *MSK_IPAR_LOG_INTPNT* .

Tuning the solver verbosity may require adjusting several parameters. It must be noticed that verbose logging is supposed to be of interest during debugging and tuning, and it is consider the default setting. When output is no more of interest, user can easily disable using *MSK_IPAR_LOG* .

Moreover, it must be understood that larger values of *MSK_IPAR_LOG* do not necessarily result in an increased output.

By default **MOSEK** will reduce the amount of log information after the first optimization on a given task. To get full log output on subsequent optimizations set *MSK_IPAR_LOG_CUT_SECOND_OPT* to zero.

PROBLEM FORMULATION AND SOLUTIONS

In this chapter we will discuss the following issues:

- The formal, mathematical formulations of the problem types that **MOSEK** can solve and their duals.

- The solution information produced by **MOSEK**.

- The infeasibility certificate produced by **MOSEK** if the problem is infeasible.

9.1 Linear Optimization

A linear optimization problem can be written as

$$
\begin{array}{lrcrcl}
\text{minimize} & & & c^T x + c^f & & \\
\text{subject to} & l^c & \leq & Ax & \leq & u^c, \\
& l^x & \leq & x & \leq & u^x,
\end{array}
\tag{9.1}
$$

where

- m is the number of constraints.

- n is the number of decision variables.

- $x \in \mathbb{R}^n$ is a vector of decision variables.

- $c \in \mathbb{R}^n$ is the linear part of the objective function.

- $A \in \mathbb{R}^{m \times n}$ is the constraint matrix.

- $l^c \in \mathbb{R}^m$ is the lower limit on the activity for the constraints.

- $u^c \in \mathbb{R}^m$ is the upper limit on the activity for the constraints.

- $l^x \in \mathbb{R}^n$ is the lower limit on the activity for the variables.

- $u^x \in \mathbb{R}^n$ is the upper limit on the activity for the variables.

A primal solution (x) is *(primal) feasible* if it satisfies all constraints in (9.1). If (9.1) has at least one primal feasible solution, then (9.1) is said to be (primal) feasible.

In case (9.1) does not have a feasible solution, the problem is said to be *(primal) infeasible*

9.1.1 Duality for Linear Optimization

Corresponding to the primal problem (9.1), there is a dual problem

$$
\begin{array}{lrcl}
\text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f & & \\
& A^T y + s_l^x - s_u^x & = & c, \\
\text{subject to} & -y + s_l^c - s_u^c & = & 0, \\
& s_l^c, s_u^c, s_l^x, s_u^x & \geq & 0.
\end{array}
\tag{9.2}
$$

If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. E.g.

$$l_j^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0 \text{ and } l_j^x \cdot (s_l^x)_j = 0.$$

This is equivalent to removing variable $(s_l^x)_j$ from the dual problem. A solution

$$(y, s_l^c, s_u^c, s_l^x, s_u^x)$$

to the dual problem is feasible if it satisfies all the constraints in (9.2). If (9.2) has at least one feasible solution, then (9.2) is *(dual) feasible*, otherwise the problem is *(dual) infeasible*.

A Primal-dual Feasible Solution

A solution

$$(x, y, s_l^c, s_u^c, s_l^x, s_u^x)$$

is denoted a *primal-dual feasible solution*, if (x) is a solution to the primal problem (9.1) and $(y, s_l^c, s_u^c, s_l^x, s_u^x)$ is a solution to the corresponding dual problem (9.2).

The Duality Gap

Let

$$(x^*, y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

be a primal-dual feasible solution, and let

$$(x^c)^* := Ax^*.$$

For a primal-dual feasible solution we define the *duality gap* as the difference between the primal and the dual objective value,

$$
\begin{aligned}
& c^T x^* + c^f - \left\{ (l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* + c^f \right\} \\
&= \sum_{i=0}^{m-1} \left[(s_l^c)_i^* ((x_i^c)^* - l_i^c) + (s_u^c)_i^* (u_i^c - (x_i^c)^*) \right] \\
&+ \sum_{j=0}^{n-1} \left[(s_l^x)_j^* (x_j - l_j^x) + (s_u^x)_j^* (u_j^x - x_j^*) \right] \geq 0
\end{aligned}
\tag{9.3}
$$

where the first relation can be obtained by transposing and multiplying the dual constraints (9.2) by x^* and $(x^c)^*$ respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

An Optimal Solution

It is well-known that a linear optimization problem has an optimal solution if and only if there exist feasible primal and dual solutions so that the duality gap is zero, or, equivalently, that the *complementarity conditions*

$$
\begin{aligned}
(s_l^c)_i^* ((x_i^c)^* - l_i^c) &= 0, & i = 0, \ldots, m-1, \\
(s_u^c)_i^* (u_i^c - (x_i^c)^*) &= 0, & i = 0, \ldots, m-1, \\
(s_l^x)_j^* (x_j^* - l_j^x) &= 0, & j = 0, \ldots, n-1, \\
(s_u^x)_j^* (u_j^x - x_j^*) &= 0, & j = 0, \ldots, n-1,
\end{aligned}
$$

are satisfied.

If (9.1) has an optimal solution and **MOSEK** solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

9.1.2 Infeasibility for Linear Optimization

Primal Infeasible Problems

If the problem (9.1) is infeasible (has no feasible solution), **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

$$
\begin{aligned}
\text{maximize} \quad & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x \\
\text{subject to} \quad & \\
& A^T y + s_l^x - s_u^x = 0, \\
& -y + s_l^c - s_u^c = 0, \\
& s_l^c, s_u^c, s_l^x, s_u^x \geq 0,
\end{aligned}
\tag{9.4}
$$

such that the objective value is strictly positive, i.e. a solution

$$
(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)
$$

to (9.4) so that

$$
(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* > 0.
$$

Such a solution implies that (9.4) is unbounded, and that its dual is infeasible. As the constraints to the dual of (9.4) are identical to the constraints of problem (9.1), we thus have that problem (9.1) is also infeasible.

Dual Infeasible Problems

If the problem (9.2) is infeasible (has no feasible solution), **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

$$
\begin{aligned}
\text{minimize} \quad & c^T x \\
\text{subject to} \quad & \hat{l}^c \leq Ax \leq \hat{u}^c, \\
& \hat{l}^x \leq x \leq \hat{u}^x,
\end{aligned}
\tag{9.5}
$$

where

$$
\hat{l}_i^c = \begin{cases} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{cases} \quad \text{and} \quad \hat{u}_i^c := \begin{cases} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{cases}
$$

and

$$
\hat{l}_j^x = \begin{cases} 0 & \text{if } l_j^x > -\infty, \\ -\infty & \text{otherwise,} \end{cases} \quad \text{and} \quad \hat{u}_j^x := \begin{cases} 0 & \text{if } u_j^x < \infty, \\ \infty & \text{otherwise,} \end{cases}
$$

such that

$$
c^T x < 0.
$$

Such a solution implies that (9.5) is unbounded, and that its dual is infeasible. As the constraints to the dual of (9.5) are identical to the constraints of problem (9.2), we thus have that problem (9.2) is also infeasible.

Primal and Dual Infeasible Case

In case that both the primal problem (9.1) and the dual problem (9.2) are infeasible, **MOSEK** will report only one of the two possible certificates — which one is not defined (**MOSEK** returns the first certificate found).

Minimalization vs. Maximalization

When the objective sense of problem (9.1) is maximization, i.e.

$$
\begin{array}{llccl}
\text{maximize} & & c^T x + c^f & & \\
\text{subject to} & l^c & \leq & Ax & \leq & u^c, \\
& l^x & \leq & x & \leq & u^x,
\end{array}
$$

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (9.2). The dual problem thus takes the form

$$
\begin{array}{ll}
\text{minimize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\
\text{subject to} &
\end{array}
$$

$$
\begin{array}{c}
A^T y + s_l^x - s_u^x = c, \\
-y + s_l^c - s_u^c = 0, \\
s_l^c, s_u^c, s_l^x, s_u^x \leq 0.
\end{array}
$$

This means that the duality gap, defined in (9.3) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective. The primal infeasibility certificate will be reported by **MOSEK** as a solution to the system

$$
\begin{array}{c}
A^T y + s_l^x - s_u^x = 0, \\
-y + s_l^c - s_u^c = 0, \\
s_l^c, s_u^c, s_l^x, s_u^x \leq 0,
\end{array}
\tag{9.6}
$$

such that the objective value is strictly negative

$$
(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* < 0.
$$

Similarly, the certificate of dual infeasibility is an x satisfying the requirements of (9.5) such that $c^T x > 0$.

9.2 Conic Quadratic Optimization

Conic quadratic optimization is an extension of linear optimization (see Section 9.1) allowing conic domains to be specified for subsets of the problem variables. A conic quadratic optimization problem can be written as

$$
\begin{array}{llccl}
\text{minimize} & & c^T x + c^f & & \\
\text{subject to} & l^c & \leq & Ax & \leq & u^c, \\
& l^x & \leq & x & \leq & u^x, \\
& & & x \in \mathcal{K}, &
\end{array}
\tag{9.7}
$$

where set $\mathcal{K}$ is a Cartesian product of convex cones, namely $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_p$. Having the domain restriction, $x \in \mathcal{K}$, is thus equivalent to

$$
x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t},
$$

where $x = (x^1, \ldots, x^p)$ is a partition of the problem variables. Please note that the n-dimensional Euclidean space $\mathbb{R}^n$ is a cone itself, so simple linear variables are still allowed.

MOSEK supports only a limited number of cones, specifically:

- The $\mathbb{R}^n$ set.

- The quadratic cone:

$$
\mathcal{Q}^n = \left\{ x \in \mathbb{R}^n : x_1 \geq \sqrt{\sum_{j=2}^{n} x_j^2} \right\}.
$$

- The rotated quadratic cone:

$$\mathcal{Q}_r^n = \left\{ x \in \mathbb{R}^n : 2x_1 x_2 \geq \sum_{j=3}^{n} x_j^2, \quad x_1 \geq 0, \quad x_2 \geq 0 \right\}.$$

Although these cones may seem to provide only limited expressive power they can be used to model a wide range of problems as demonstrated in *[MOSEKApS12]*.

9.2.1 Duality for Conic Quadratic Optimization

The dual problem corresponding to the conic quadratic optimization problem (9.7) is given by

$$
\begin{aligned}
\text{maximize} \quad & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\
\text{subject to} \quad & \\
& A^T y + s_l^x - s_u^x + s_n^x = c \\
& -y + s_l^c - s_u^c = 0, \\
& s_l^c, s_u^c, s_l^x, s_u^x \geq 0, \\
& s_n^x \in \mathcal{K}^*,
\end{aligned}
\tag{9.8}
$$

where the dual cone $\mathcal{K}^*$ is a Cartesian product of the cones

$$\mathcal{K}^* = \mathcal{K}_1^* \times \cdots \times \mathcal{K}_p^*,$$

where each $\mathcal{K}_t^*$ is the dual cone of $\mathcal{K}_t$. For the cone types **MOSEK** can handle, the relation between the primal and dual cone is given as follows:

- The $\mathbb{R}^n$ set:

$$\mathcal{K}_t = \mathbb{R}^{n_t} \quad \Leftrightarrow \quad \mathcal{K}_t^* = \left\{ s \in \mathbb{R}^{n_t} : \quad s = 0 \right\}.$$

- The quadratic cone:

$$\mathcal{K}_t = \mathcal{Q}^{n_t} \quad \Leftrightarrow \quad \mathcal{K}_t^* = \mathcal{Q}^{n_t} = \left\{ s \in \mathbb{R}^{n_t} : s_1 \geq \sqrt{\sum_{j=2}^{n_t} s_j^2} \right\}.$$

- The rotated quadratic cone:

$$\mathcal{K}_t = \mathcal{Q}_r^{n_t} \quad \Leftrightarrow \quad \mathcal{K}_t^* = \mathcal{Q}_r^{n_t} = \left\{ s \in \mathbb{R}^{n_t} : 2s_1 s_2 \geq \sum_{j=3}^{n_t} s_j^2, \quad s_1 \geq 0, \quad s_2 \geq 0 \right\}.$$

Please note that the dual problem of the dual problem is identical to the original primal problem.

9.2.2 Infeasibility for Conic Quadratic Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of infeasibility. This works exactly as for linear problems (see Section *9.1.2*).

Primal Infeasible Problems

If the problem (9.7) is infeasible, **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the problem

$$
\begin{aligned}
\text{maximize} \quad & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x \\
\text{subject to} \quad & \\
& A^T y + s_l^x - s_u^x + s_n^x && = && 0, \\
& -y + s_l^c - s_u^c && = && 0, \\
& s_l^c, s_u^c, s_l^x, s_u^x && \geq && 0, \\
& s_n^x \in \mathcal{K}^*,
\end{aligned}
$$

such that the objective value is strictly positive.

Dual infeasible problems

If the problem (9.8) is infeasible, **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

$$
\begin{array}{llcccl}
\text{minimize} & & & c^T x & & \\
\text{subject to} & \hat{l}^c & \leq & Ax & \leq & \hat{u}^c, \\
& \hat{l}^x & \leq & x & \leq & \hat{u}^x, \\
& & & x \in \mathcal{K},
\end{array}
$$

where

$$
\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}
$$

and

$$
\hat{l}_j^x = \left\{ \begin{array}{ll} 0 & \text{if } l_j^x > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_j^x := \left\{ \begin{array}{ll} 0 & \text{if } u_j^x < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}
$$

such that the objective value is strictly negative.

9.3 Semidefinite Optimization

Semidefinite optimization is an extension of conic quadratic optimization (see Section *9.2*) allowing positive semidefinite matrix variables to be used in addition to the usual scalar variables. A semidefinite optimization problem can be written as

$$
\begin{array}{llccll}
\text{minimize} & & \sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \langle \overline{C}_j, \overline{X}_j \rangle + c^f & & \\
\text{subject to} & l_i^c \leq & \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \langle \overline{A}_{ij}, \overline{X}_j \rangle & \leq & u_i^c, & i = 0, \ldots, m-1 \\
& l_j^x \leq & x_j & \leq & u_j^x, & j = 0, \ldots, n-1 \\
& & x \in \mathcal{K}, \overline{X}_j \in \mathcal{S}_+^{r_j}, & & & j = 0, \ldots, p-1
\end{array} \tag{9.9}
$$

where the problem has p symmetric positive semidefinite variables $\overline{X}_j \in \mathcal{S}_+^{r_j}$ of dimension r_j with symmetric coefficient matrices $\overline{C}_j \in \mathcal{S}^{r_j}$ and $\overline{A}_{i,j} \in \mathcal{S}^{r_j}$. We use standard notation for the matrix inner product, i.e., for $U, V \in \mathbb{R}^{m \times n}$ we have

$$
\langle U, V \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} U_{ij} V_{ij}.
$$

With semidefinite optimization we can model a wide range of problems as demonstrated in [MOSEKApS12].

9.3.1 Duality for Semidefinite Optimization

The dual problem corresponding to the semidefinite optimization problem (9.9) is given by

$$
\begin{array}{ll}
\text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\
\text{subject to} & \\
& c - A^T y + s_u^x - s_l^x = s_n^x, \\
& \overline{C}_j - \sum_{i=0}^m y_i \overline{A}_{ij} = \overline{S}_j, \qquad\qquad\qquad j = 0, \ldots, p-1 \\
& s_l^c - s_u^c = y, \\
& s_l^c, s_u^c, s_l^x, s_u^x \geq 0, \\
& s_n^x \in \mathcal{K}^*, \quad \overline{S}_j \in \mathcal{S}_+^{r_j}, \qquad\qquad\qquad j = 0, \ldots, p-1
\end{array} \tag{9.10}
$$

where $A \in \mathbb{R}^{m \times n}$, $A_{ij} = a_{ij}$, which is similar to the dual problem for conic quadratic optimization (see Section *9.2.1*), except for the addition of dual constraints

$$\left(\overline{C}_j - \sum_{i=0}^{m} y_i \overline{A}_{ij} \right) \in \mathcal{S}_+^{r_j}.$$

Note that the dual of the dual problem is identical to the original primal problem.

9.3.2 Infeasibility for Semidefinite Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of the infeasibility. This works exactly as for linear problems (see Section *9.1.2*).

Primal Infeasible Problems

If the problem (9.9) is infeasible, **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is a certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the problem

$$
\begin{aligned}
\text{maximize} \quad & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x \\
\text{subject to} \quad & \\
& A^T y + s_l^x - s_u^x + s_n^x = 0, \\
& \sum_{i=0}^{m-1} y_i \overline{A}_{ij} + \overline{S}_j = 0, \qquad\qquad j = 0, \dots, p-1 \\
& -y + s_l^c - s_u^c = 0, \\
& s_l^c, s_u^c, s_l^x, s_u^x \geq 0, \\
& s_n^x \in \mathcal{K}^*, \quad \overline{S}_j \in \mathcal{S}_+^{r_j}, \qquad\qquad j = 0, \dots, p-1
\end{aligned}
$$

such that the objective value is strictly positive.

Dual Infeasible Problems

If the problem (9.10) is infeasible, **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

$$
\begin{aligned}
\text{minimize} \quad & \sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle \\
\text{subject to} \quad \hat{l}_i^c \leq\; & \sum_{j=1}^{n} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq \hat{u}_i^c, \quad i = 0, \dots, m-1 \\
\hat{l}^x \leq\; & \qquad\qquad x \qquad\qquad \leq \hat{u}^x, \\
& x \in \mathcal{K}, \quad \overline{X}_j \in \mathcal{S}_+^{r_j}, \qquad\qquad j = 0, \dots, p-1
\end{aligned}
$$

where

$$
\hat{l}_i^c = \begin{cases} 0 & \text{if } l_i^c >; -\infty, \\ -\infty & \text{otherwise,} \end{cases} \quad \text{and} \quad \hat{u}_i^c := \begin{cases} 0 & \text{if } u_i^c <; \infty, \\ \infty & \text{otherwise,} \end{cases}
$$

and

$$
\hat{l}_j^x = \begin{cases} 0 & \text{if } l_j^x >; -\infty, \\ -\infty & \text{otherwise,} \end{cases} \quad \text{and} \quad \hat{u}_j^x := \begin{cases} 0 & \text{if } u_j^x <; \infty, \\ \infty & \text{otherwise,} \end{cases}
$$

such that the objective value is strictly negative.

9.4 Quadratic and Quadratically Constrained Optimization

A convex quadratic and quadratically constrained optimization problem is an optimization problem of the form

$$
\begin{array}{llrcll}
\text{minimize} & & \frac{1}{2}x^T Q^o x + c^T x + c^f & & & \\
\text{subject to} & l_k^c \le & \frac{1}{2}x^T Q^k x + \sum_{j=0}^{n-1} a_{kj} x_j & \le & u_k^c, & k = 0, \ldots, m-1, \\
& l_j^x \le & x_j & \le & u_j^x, & j = 0, \ldots, n-1,
\end{array}
\tag{9.11}
$$

where Q^o and all Q^k are symmetric matrices. Moreover for convexity, Q^o must be a positive semidefinite matrix and Q^k must satisfy

$$
\begin{array}{rcl}
-\infty < l_k^c & \Rightarrow & Q^k \text{ is negative semidefinite,} \\
u_k^c < \infty & \Rightarrow & Q^k \text{ is positive semidefinite,} \\
-\infty < l_k^c \le u_k^c < \infty & \Rightarrow & Q^k = 0.
\end{array}
$$

The convexity requirement is very important and **MOSEK** checks whether it is fullfiled.

9.4.1 A Recommendation

Any convex quadratic optimization problem can be reformulated as a conic quadratic optimization problem, see [MOSEKApS12] and in particular [And13]. In fact **MOSEK** does such conversion internally as a part of the solution process for the following reasons:

- the conic optimizer is numerically more robust than the one for quadratic problems.
- the conic optimizer is usually faster because quadratic cones are simpler than quadratic functions, even though the conic reformulation usually has more constraints and variables than the original quadratic formulation.
- it is easy to dualize the conic formuation if deemed worthwhile potentially leading to (huge) computational savings.

However, instead of relying on the automatic reformulation we recommend to formulate the problem as conic problem from scratch because:

- it saves the computational overhead of the reformulation including the convexity check. A conic problem is convex by construction and hence no convexity check is neeeded for conic problems.
- usually the modeller can do a better reformulation than the automatic method because the modeller can exploit the knowledge of what is being modelled.

To summarize we recommend to formulate quadratic problems and in particular quadratically constrained problems directly in conic form.

9.4.2 Duality for Quadratic and Quadratically Constrained Optimization

The dual problem corresponding to the quadratic and quadratically constrained optimization problem (9.11) is given by

$$
\begin{array}{ll}
\text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + \frac{1}{2}x^T \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x + c^f \\
\text{subject to} &
\end{array}
$$

$$
\begin{array}{rcl}
A^T y + s_l^x - s_u^x + \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x & = & c, \\
-y + s_l^c - s_u^c & = & 0, \\
s_l^c, s_u^c, s_l^x, s_u^x & \ge & 0.
\end{array}
\tag{9.12}
$$

The dual problem is related to the dual problem for linear optimization (see Section 9.1.1), but depends on the variable x which in general can not be eliminated. In the solutions reported by **MOSEK**, the value of x is the same for the primal problem (9.11) and the dual problem (9.12).

9.4.3 Infeasibility for Quadratic and Quadratically Constrained Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of infeasibility. This works exactly as for linear problems (see Section *9.1.2*).

Primal Infeasible Problems

If the problem (9.11) with all $Q^k = 0$ is infeasible, **MOSEK** will report a certificate of primal infeasibility. As the constraints are the same as for a linear problem, the certificate of infeasibility is the same as for linear optimization (see Section *9.1.2.1*).

Dual Infeasible Problems

If the problem (9.12) with all $Q^k = 0$ is infeasible, **MOSEK** will report a certificate of dual infeasibility. The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

$$
\begin{array}{rcccc}
\text{minimize} & & c^T x & & \\
\text{subject to} & \hat{l}^c & \leq & Ax & \leq & \hat{u}^c, \\
& 0 & \leq & Q^o x & \leq & 0, \\
& \hat{l}^x & \leq & x & \leq & \hat{u}^x,
\end{array}
$$

where

$$
\hat{l}^c_i = \left\{ \begin{array}{ll} 0 & \text{if } l^c_i > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}^c_i := \left\{ \begin{array}{ll} 0 & \text{if } u^c_i < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}
$$

and

$$
\hat{l}^x_j = \left\{ \begin{array}{ll} 0 & \text{if } l^x_j > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}^x_j := \left\{ \begin{array}{ll} 0 & \text{if } u^x_j < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}
$$

such that the objective value is strictly negative.

9.5 General Convex Optimization

MOSEK is capable of solving smooth (twice differentiable) convex nonlinear optimization problems of the form

$$
\begin{array}{rccccc}
\text{minimize} & & & f(x) + c^T x + c^f & & \\
\text{subject to} & l^c & \leq & g(x) + Ax & \leq & u^c, \\
& l^x & \leq & x & \leq & u^x,
\end{array}
$$

where

- m is the number of constraints.

- n is the number of decision variables.

- $x \in \mathbb{R}^n$ is a vector of decision variables.

- $c \in \mathbb{R}^n$ is the linear part objective function.

- $A \in \mathbb{R}^{m \times n}$ is the constraint matrix.

- $l^c \in \mathbb{R}^m$ is the lower limit on the activity for the constraints.

- $u^c \in \mathbb{R}^m$ is the upper limit on the activity for the constraints.

- $l^x \in \mathbb{R}^n$ is the lower limit on the activity for the variables.

- $u^x \in \mathbb{R}^n$ is the upper limit on the activity for the variables.

- $f : \mathbb{R}^n \to \mathbb{R}$ is a nonlinear function.

- $g : \mathbb{R}^n \to \mathbb{R}^m$ is a nonlinear vector function.

This means that the i-th constraint has the form

$$l_i^c \leq g_i(x) + \sum_{j=1}^n a_{ij} x_j \leq u_i^c.$$

The linear term Ax is included in $g(x)$ since it can be handled much more efficiently as a separate entity when optimizing.

The nonlinear functions f and g must be smooth in all $x \in [l^x; u^x]$. Moreover, $f(x)$ must be a convex function and $g_i(x)$ must satisfy

$$
\begin{aligned}
-\infty < l_i^c &\Rightarrow & g_i(x) \text{ is concave,} \\
u_i^c < \infty &\Rightarrow & g_i(x) \text{ is convex,} \\
-\infty < l_i^c \leq u_i^c < \infty &\Rightarrow & g_i(x) = 0.
\end{aligned}
$$

9.5.1 Duality for General convex Optimization

Similar to the linear case, **MOSEK** reports dual information in the general nonlinear case. Indeed in this case the Lagrange function is defined by

$$
\begin{aligned}
L(x, s_l^c, s_u^c, s_l^x, s_u^x) \quad := \quad & f(x) + c^T x + c^f \\
& -(s_l^c)^T (g(x) + Ax - l^c) - (s_u^c)^T (u^c - g(x) - Ax) \\
& -(s_l^x)^T (x - l^x) - (s_u^x)^T (u^x - x),
\end{aligned}
$$

and the dual problem is given by

$$
\begin{aligned}
\text{maximize} \quad & L(x, s_l^c, s_u^c, s_l^x, s_u^x) \\
\text{subject to} \quad & \nabla_x L(x, s_l^c, s_u^c, s_l^x, s_u^x)^T = 0, \\
& s_l^c, s_u^c, s_l^x, s_u^x \geq 0,
\end{aligned}
$$

which is equivalent to

$$
\begin{aligned}
\text{maximize} \quad & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\
& + f(x) - g(x)^T y - (\nabla f(x)^T - \nabla g(x)^T y)^T x \\
\text{subject to} \quad & A^T y + s_l^x - s_u^x - (\nabla f(x)^T - \nabla g(x)^T y) = c, \\
& -y + s_l^c - s_u^c = 0, \\
& s_l^c, s_u^c, s_l^x, s_u^x \geq 0.
\end{aligned}
$$

In this context we use the following definition for scalar functions

$$\nabla f(x) = \left[\frac{\partial f(x)}{\partial x_1}, \ldots, \frac{\partial f(x)}{\partial x_n} \right],$$

and accordingly for vector functions

$$\nabla g(x) = \begin{bmatrix} \nabla g_1(x) \\ \vdots \\ \nabla g_m(x) \end{bmatrix}.$$

TEN

THE OPTIMIZERS FOR CONTINUOUS PROBLEMS

The most essential part of **MOSEK** are the optimizers. This chapter describes the optimizers for the class of *continuous problems* without integer variables, that is:

- linear problems,
- conic problems (quadratic and semidefinite),
- general convex problems.

MOSEK offers an interior-point optimizer for each class of problems and also a simplex optimizer for linear problems. The structure of a successful optimization process is roughly:

- **Presolve**
  1. *Elimination*: Reduce the size of the problem.
  2. *Dualizer*: Choose whether to solve the primal or the dual form of the problem.
  3. *Scaling*: Scale the problem for better numerical stability.
- **Optimization**
  1. *Optimize*: Solve the problem using selected method.
  2. *Terminate*: Stop the optimization when specific termination criteria have been met.
  3. *Report*: Return the solution or an infeasibility certificate.

The preprocessing stage is transparent to the user, but useful to know about for tuning purposes. The purpose of the preprocessing steps is to make the actual optimization more efficient and robust. We discuss the details of the above steps in the following sections.

10.1 Presolve

Before an optimizer actually performs the optimization the problem is preprocessed using the so-called presolve. The purpose of the presolve is to

1. remove redundant constraints,
2. eliminate fixed variables,
3. remove linear dependencies,
4. substitute out (implied) free variables, and
5. reduce the size of the optimization problem in general.

After the presolved problem has been optimized the solution is automatically postsolved so that the returned solution is valid for the original problem. Hence, the presolve is completely transparent. For further details about the presolve phase, please see *[AA95]* and *[AGMX96]*.

It is possible to fine-tune the behavior of the presolve or to turn it off entirely. If presolve consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This is

done by setting the parameter `MSK_IPAR_PRESOLVE_USE` to `"MSK_PRESOLVE_MODE_OFF"`. The two most time-consuming steps of the presolve are

- the eliminator, and

- the linear dependency check.

Therefore, in some cases it is worthwhile to disable one or both of these.

Numerical issues in the presolve

During the presolve the problem is reformulated so that it hopefully solves faster. However, in rare cases the presolved problem may be harder to solve then the original problem. The presolve may also be infeasible although the original problem is not. If it is suspected that presolved problem is much harder to solve than the original, we suggest to first turn the eliminator off by setting the parameter `MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_NUM_TRIES` to 0. If that does not help, then trying to turn entire presolve off may help.

Since all computations are done in finite precision, the presolve employs some tolerances when concluding a variable is fixed or a constraint is redundant. If it happens that **MOSEK** incorrectly concludes a problem is primal or dual infeasible, then it is worthwhile to try to reduce the parameters `MSK_DPAR_PRESOLVE_TOL_X` and `MSK_DPAR_PRESOLVE_TOL_S`. However, if reducing the parameters actually helps then this should be taken as an indication that the problem is badly formulated.

Eliminator

The purpose of the eliminator is to eliminate free and implied free variables from the problem using substitution. For instance, given the constraints

$$\begin{array}{rcl} y & = & \sum_j x_j, \\ y, x & \geq & 0, \end{array}$$

y is an implied free variable that can be substituted out of the problem, if deemed worthwhile. If the eliminator consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This can be done by setting the parameter `MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_NUM_TRIES` to 0. In rare cases the eliminator may cause that the problem becomes much hard to solve.

Linear dependency checker

The purpose of the linear dependency check is to remove linear dependencies among the linear equalities. For instance, the three linear equalities

$$\begin{array}{rcl} x_1 + x_2 + x_3 & = & 1, \\ x_1 + 0.5x_2 & = & 0.5, \\ 0.5x_2 + x_3 & = & 0.5. \end{array}$$

contain exactly one linear dependency. This implies that one of the constraints can be dropped without changing the set of feasible solutions. Removing linear dependencies is in general a good idea since it reduces the size of the problem. Moreover, the linear dependencies are likely to introduce numerical problems in the optimization phase. It is best practice to build models without linear dependencies, but that is not always easy for the user to control. If the linear dependencies are removed at the modelling stage, the linear dependency check can safely be disabled by setting the parameter `MSK_IPAR_PRESOLVE_LINDEP_USE` to `"MSK_OFF"`.

Dualizer

All linear, conic, and convex optimization problems have an equivalent dual problem associated with them. **MOSEK** has built-in heuristics to determine if it is more efficient to solve the primal or dual

problem. The form (primal or dual) is displayed in the **MOSEK** log and available as an information item from the solver. Should the internal heuristics not choose the most efficient form of the problem it may be worthwhile to set the dualizer manually by setting the parameters:

- *MSK_IPAR_INTPNT_SOLVE_FORM*: In case of the interior-point optimizer.

- *MSK_IPAR_SIM_SOLVE_FORM*: In case of the simplex optimizer.

Note that currently only linear and conic quadratic problems may be automatically dualized.

Scaling

Problems containing data with large and/or small coefficients, say $1.0e + 9$ or $1.0e - 7$, are often hard to solve. Significant digits may be truncated in calculations with finite precision, which can result in the optimizer relying on inaccurate data. Since computers work in finite precision, extreme coefficients should be avoided. In general, data around the same *order of magnitude* is preferred, and we will refer to a problem, satisfying this loose property, as being *well-scaled*. If the problem is not well scaled, **MOSEK** will try to scale (multiply) constraints and variables by suitable constants. **MOSEK** solves the scaled problem to improve the numerical properties.

The scaling process is transparent, i.e. the solution to the original problem is reported. It is important to be aware that the optimizer terminates when the termination criterion is met on the scaled problem, therefore significant primal or dual infeasibilities may occur after unscaling for badly scaled problems. The best solution of this issue is to reformulate the problem, making it better scaled.

By default **MOSEK** heuristically chooses a suitable scaling. The scaling for interior-point and simplex optimizers can be controlled with the parameters *MSK_IPAR_INTPNT_SCALING* and *MSK_IPAR_SIM_SCALING* respectively.

10.2 Using Multiple Threads in an Optimizer

Multithreading in interior-point optimizers

The interior-point optimizers in **MOSEK** have been parallelized. This means that if you solve linear, quadratic, conic, or general convex optimization problem using the interior-point optimizer, you can take advantage of multiple CPU's. By default **MOSEK** will automatically select the number of threads to be employed when solving the problem. However, the maximum number of threads employed can be changed by setting the parameter *MSK_IPAR_NUM_THREADS*. This should never exceed the number of cores on the computer.

The speed-up obtained when using multiple threads is highly problem and hardware dependent, and consequently, it is advisable to compare single threaded and multi threaded performance for the given problem type to determine the optimal settings. For small problems, using multiple threads is not be worthwhile and may even be counter productive because of the additional coordination overhead. Therefore, it may be advantageous to disable multithreading using the parameter *MSK_IPAR_INTPNT_MULTI_THREAD*.

The interior-point optimizer parallelizes big tasks such linear algebra computations.

Thread Safety

The **MOSEK** API is thread-safe provided that a task is only modified or accessed from one thread at any given time. Also accessing two or more separate tasks from threads at the same time is safe. Sharing an environment between threads is safe.

Determinism

The optimizers are run-to-run deterministic which means if a problem is solved twice on the same computer using the same parameter setting and exactly the same input then exactly the same results is obtained. One restriction is that no time limits must be imposed because the time taken to perform an operation on a computer is dependent on many factors such as the current workload.

10.3 Linear Optimization

10.3.1 Optimizer Selection

Two different types of optimizers are available for linear problems: The default is an interior-point method, and the alternative is the simplex method (primal or dual). The optimizer can be selected using the parameter `MSK_IPAR_OPTIMIZER`.

The Interior-point or the Simplex Optimizer?

Given a linear optimization problem, which optimizer is the best: the simplex or the interior-point optimizer? It is impossible to provide a general answer to this question. However, the interior-point optimizer behaves more predictably: it tends to use between 20 and 100 iterations, almost independently of problem size, but cannot perform warm-start. On the other hand the simplex method can take advantage of an initial solution, but is less predictable from cold-start. The interior-point optimizer is used by default.

The Primal or the Dual Simplex Variant?

MOSEK provides both a primal and a dual simplex optimizer. Predicting which simplex optimizer is faster is impossible, however, in recent years the dual optimizer has seen several algorithmic and computational improvements, which, in our experience, make it faster on average than the primal version. Still, it depends much on the problem structure and size. Setting the `MSK_IPAR_OPTIMIZER` parameter to `"MSK_OPTIMIZER_FREE_SIMPLEX"` instructs **MOSEK** to choose one of the simplex variants automatically.

To summarize, if you want to know which optimizer is faster for a given problem type, it is best to try all the options.

10.3.2 The Interior-point Optimizer

The purpose of this section is to provide information about the algorithm employed in the **MOSEK** interior-point optimizer for linear problems and about its termination criteria.

The homogeneous primal-dual problem

In order to keep the discussion simple it is assumed that **MOSEK** solves linear optimization problems of standard form

$$
\begin{array}{lll}
\text{minimize} & c^T x \\
\text{subject to} & Ax & = & b, \\
& x \geq 0.
\end{array}
\tag{10.1}
$$

This is in fact what happens inside **MOSEK**; for efficiency reasons **MOSEK** converts the problem to standard form before solving, then converts it back to the input form when reporting the solution.

Since it is not known beforehand whether problem (10.1) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason why **MOSEK** solves the so-called homogeneous model

$$
\begin{array}{rcl}
Ax - b\tau & = & 0, \\
A^T y + s - c\tau & = & 0, \\
-c^T x + b^T y - \kappa & = & 0, \\
x, s, \tau, \kappa & \geq & 0,
\end{array}
\tag{10.2}
$$

where y and s correspond to the dual variables in (10.1), and τ and κ are two additional scalar variables. Note that the homogeneous model (10.2) always has solution since

$$
(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)
$$

is a solution, although not a very interesting one. Any solution

$$
(x^*, y^*, s^*, \tau^*, \kappa^*)
$$

to the homogeneous model (10.2) satisfies

$$
x_j^* s_j^* = 0 \text{ and } \tau^* \kappa^* = 0.
$$

Moreover, there is always a solution that has the property $\tau^* + \kappa^* > 0$.

First, assume that $\tau^* > 0$. It follows that

$$
\begin{array}{rcl}
A \frac{x^*}{\tau^*} & = & b, \\
A^T \frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\
-c^T \frac{x^*}{\tau^*} + b^T \frac{y^*}{\tau^*} & = & 0, \\
x^*, s^*, \tau^*, \kappa^* & \geq & 0.
\end{array}
$$

This shows that $\frac{x^*}{\tau^*}$ is a primal optimal solution and $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$ is a dual optimal solution; this is reported as the optimal interior-point solution since

$$
(x, y, s) = \left\{ \frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*} \right\}
$$

is a primal-dual optimal solution (see Section *9.1* for the mathematical background on duality and optimality).

On other hand, if $\kappa^* > 0$ then

$$
\begin{array}{rcl}
Ax^* & = & 0, \\
A^T y^* + s^* & = & 0, \\
-c^T x^* + b^T y^* & = & \kappa^*, \\
x^*, s^*, \tau^*, \kappa^* & \geq & 0.
\end{array}
$$

This implies that at least one of

$$
c^T x^* < 0
\tag{10.3}
$$

or

$$
b^T y^* > 0
\tag{10.4}
$$

is satisfied. If (10.3) is satisfied then x^* is a certificate of dual infeasibility, whereas if (10.4) is satisfied then y^* is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm *[And09]*.

Interior-point Termination Criterion

For efficiency reasons it is not practical to solve the homogeneous model exactly. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In the k-th iteration of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to homogeneous model is generated, where

$$x^k, s^k, \tau^k, \kappa^k > 0.$$

Optimal case

Whenever the trial solution satisfies the criterion

$$
\begin{aligned}
\left\| A \tfrac{x^k}{\tau^k} - b \right\|_\infty &\leq \epsilon_p (1 + \|b\|_\infty), \\
\left\| A^T \tfrac{y^k}{\tau^k} + \tfrac{s^k}{\tau^k} - c \right\|_\infty &\leq \epsilon_d (1 + \|c\|_\infty), \text{ and} \\
\min\left(\tfrac{(x^k)^T s^k}{(\tau^k)^2}, \left| \tfrac{c^T x^k}{\tau^k} - \tfrac{b^T y^k}{\tau^k} \right| \right) &\leq \epsilon_g \max\left(1, \tfrac{\min\left(|c^T x^k|, |b^T y^k| \right)}{\tau^k} \right),
\end{aligned}
\tag{10.5}
$$

the interior-point optimizer is terminated and

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is reported as the primal-dual optimal solution. The interpretation of (10.5) is that the optimizer is terminated if

- $\tfrac{x^k}{\tau^k}$ is approximately primal feasible,

- $\left\{ \tfrac{y^k}{\tau^k}, \tfrac{s^k}{\tau^k} \right\}$ is approximately dual feasible, and

- the duality gap is almost zero.

Dual infeasibility certificate

On the other hand, if the trial solution satisfies

$$-\epsilon_i c^T x^k > \frac{\|c\|_\infty}{\max\left(1, \|b\|_\infty \right)} \left\| A x^k \right\|_\infty$$

then the problem is declared dual infeasible and x^k is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows: First assume that $\left\| A x^k \right\|_\infty = 0$; then x^k is an exact certificate of dual infeasibility. Next assume that this is not the case, i.e.

$$\left\| A x^k \right\|_\infty > 0,$$

and define

$$\bar{x} := \epsilon_i \frac{\max\left(1, \|b\|_\infty \right)}{\left\| A x^k \right\|_\infty \|c\|_\infty} x^k.$$

It is easy to verify that

$$\left\| A \bar{x} \right\|_\infty = \epsilon_i \frac{\max\left(1, \|b\|_\infty \right)}{\|c\|_\infty} \text{ and } -c^T \bar{x} > 1,$$

which shows $\bar{x}$ is an approximate certificate of dual infeasibility, where ε_i controls the quality of the approximation. A smaller value means a better approximation.

Primal infeasibility certificate

Finally, if

$$\epsilon_i b^T y^k > \frac{\|b\|_\infty}{\max\left(1, \|c\|_\infty\right)} \left\|A^T y^k + s^k\right\|_\infty$$

then y^k is reported as a certificate of primal infeasibility.

Adjusting optimality criteria and near optimality

It is possible to adjust the tolerances ε_p, ε_d, ε_g and ε_i using parameters; see table for details.

Table 10.1: Parameters employed in termination criterion

ToleranceParameter	name
ε_p	MSK_DPAR_INTPNT_TOL_PFEAS
ε_d	MSK_DPAR_INTPNT_TOL_DFEAS
ε_g	MSK_DPAR_INTPNT_TOL_REL_GAP
ε_i	MSK_DPAR_INTPNT_TOL_INFEAS

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (10.5) reveals that the quality of the solution depends on $\|b\|_\infty$ and $\|c\|_\infty$; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate *[And09]*. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances, ε_p, ε_d, ε_g and ε_i, have to be relaxed together to achieve an effect.

In some cases the interior-point method terminates having found a solution not too far from meeting the optimality condition (10.5). A solution is defined as *near optimal* if scaling the termination tolerances ε_p, ε_d, ε_g and ε_g by the same factor $\varepsilon_n \in [1.0, +\infty]$ makes the condition (10.5) satisfied. A near optimal solution is therefore of lower quality but still potentially valuable. If for instance the solver stalls, i.e. it can make no more significant progress towards the optimal solution, a near optimal solution could be available and be good enough for the user. Near infeasibility certificates are defined similarly. The value of ε_n can be adjusted with the parameter MSK_DPAR_INTPNT_CO_TOL_NEAR_REL.

The basis identification discussed in Section *10.3.2.4* requires an optimal solution to work well; hence basis identification should be turned off if the termination criterion is relaxed.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

Basis Identification

An interior-point optimizer does not return an optimal basic solution unless the problem has a unique primal and dual optimal solution. Therefore, the interior-point optimizer has an optional post-processing step that computes an optimal basic solution starting from the optimal interior-point solution. More information about the basis identification procedure may be found in *[AY96]*. In the following we provide an overall idea of the procedure.

There are some cases in which a basic solution could be more valuable:

- a basic solution is often more accurate than an interior-point solution,

- a basic solution can be used to warm-start the simplex algorithm in case of reoptimization,

- a basic solution is in general more sparse, i.e. more variables are fixed to zero. This is particularly appealing when solving continuous relaxations of mixed integer problems, as well as in all applications in which sparser solutions are preferred.

To illustrate how the basis identification routine works, we use the following trivial example:

$$\begin{array}{llll} \text{minimize} & x + y & & \\ \text{subject to} & x + y & = & 1, \\ & x, y \geq 0. & & \end{array}$$

It is easy to see that all feasible solutions are also optimal. In particular, there are two basic solutions, namely

$$\begin{array}{lll} (x_1^*, y_1^*) & = & (1, 0), \\ (x_2^*, y_2^*) & = & (0, 1). \end{array}$$

The interior point algorithm will actually converge to the center of the optimal set, i.e. to $(x^*, y^*) = (1/2, 1/2)$ (to see this in **MOSEK** deactivate *Presolve*).

In practice, when the algorithm gets close to the optimal solution, it is possible to construct in polynomial time an initial basis for the simplex algorithm from the current interior point solution. This basis is used to warm-start the simplex algorithm that will provide the optimal basic solution. In most cases the constructed basis is optimal, or very few iterations are required by the simplex algorithm to make it optimal and hence the final *clean-up* phase be short. However, for some cases of ill-conditioned problems the additional simplex clean up phase may take of lot a time.

By default **MOSEK** performs a basis identification. However, if a basic solution is not needed, the basis identification procedure can be turned off. The parameters

- *MSK_IPAR_INTPNT_BASIS* ,

- *MSK_IPAR_BI_IGNORE_MAX_ITER* , and

- *MSK_IPAR_BI_IGNORE_NUM_ERROR*

control when basis identification is performed.

The type of simplex algorithm to be used (primal/dual) can be tuned with the parameter *MSK_IPAR_BI_CLEAN_OPTIMIZER* , and the maximum number of iterations can be set with *MSK_IPAR_BI_MAX_ITERATIONS* .

Finally, it should be mentioned that there is no guarantee on which basic solution will be returned.

The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer  - threads             : 1
Optimizer  - solved problem      : the dual
Optimizer  - Constraints         : 2
Optimizer  - Cones               : 0
Optimizer  - Scalar variables    : 6                conic            : 0
Optimizer  - Semi-definite variables: 0             scalarized       : 0
Factor     - setup time          : 0.00             dense det. time  : 0.00
Factor     - ML order time       : 0.00             GP order time    : 0.00
Factor     - nonzeros before factor : 3             after factor     : 3
Factor     - dense dim.          : 0                flops            : 7.00e+001
ITE PFEAS    DFEAS    GFEAS    PRSTATUS  POBJ              DOBJ              MU       TIME
0   1.0e+000 8.6e+000 6.1e+000 1.00e+000 0.000000000e+000 -2.208000000e+003 1.0e+000 0.00
1   1.1e+000 2.5e+000 1.6e-001 0.00e+000 -7.901380925e+003 -7.394611417e+003 2.5e+000 0.00
2   1.4e-001 3.4e-001 2.1e-002 8.36e-001 -8.113031650e+003 -8.055866001e+003 3.3e-001 0.00
3   2.4e-002 5.8e-002 3.6e-003 1.27e+000 -7.777530698e+003 -7.766471080e+003 5.7e-002 0.01
4   1.3e-004 3.2e-004 2.0e-005 1.08e+000 -7.668323435e+003 -7.668207177e+003 3.2e-004 0.01
5   1.3e-008 3.2e-008 2.0e-009 1.00e+000 -7.668000027e+003 -7.668000015e+003 3.2e-008 0.01
6   1.3e-012 3.2e-012 2.0e-013 1.00e+000 -7.667999994e+003 -7.667999994e+003 3.2e-012 0.01
```

The first line displays the number of threads used by the optimizer and the second line tells that the optimizer chose to solve the dual problem rather than the primal problem. The next line displays the

problem dimensions as seen by the optimizer, and the `Factor...` lines show various statistics. This is followed by the iteration log.

Using the same notation as in Section *10.3.2* the columns of the iteration log have the following meaning:

- `ITE`: Iteration index k.

- `PFEAS`: $\left\| Ax^k - b\tau^k \right\|_\infty$. The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.

- `DFEAS`: $\left\| A^T y^k + s^k - c\tau^k \right\|_\infty$. The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.

- `GFEAS`: $\left| -c^T x^k + b^T y^k - \kappa^k \right|$. The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.

- `PRSTATUS`: This number converges to 1 if the problem has an optimal solution whereas it converges to -1 if that is not the case.

- `POBJ`: $c^T x^k / \tau^k$. An estimate for the primal objective value.

- `DOBJ`: $b^T y^k / \tau^k$. An estimate for the dual objective value.

- `MU`: $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$. The numbers in this column should always converge to zero.

- `TIME`: Time spent since the optimization started.

10.3.3 The Simplex Optimizer

An alternative to the interior-point optimizer is the simplex optimizer. The simplex optimizer uses a different method that allows exploiting an initial guess for the optimal solution to reduce the solution time. Depending on the problem it may be faster or slower to use an initial guess; see section *10.3.1* for a discussion. **MOSEK** provides both a primal and a dual variant of the simplex optimizer.

Simplex Termination Criterion

The simplex optimizer terminates when it finds an optimal basic solution or an infeasibility certificate. A basic solution is optimal when it is primal and dual feasible; see Section *9.1* for a definition of the primal and dual problem. Due to the fact that computations are performed in finite precision **MOSEK** allows violations of primal and dual feasibility within certain tolerances. The user can control the allowed primal and dual tolerances with the parameters *MSK_DPAR_BASIS_TOL_X* and *MSK_DPAR_BASIS_TOL_S* .

Setting the parameter *MSK_IPAR_OPTIMIZER* to *"MSK_OPTIMIZER_FREE_SIMPLEX"* instructs **MOSEK** to select automatically between the primal and the dual simplex optimizers. Hence, **MOSEK** tries to choose the best optimizer for the given problem and the available solution. The same parameter can also be used to force one of the variants.

Starting From an Existing Solution

When using the simplex optimizer it may be possible to reuse an existing solution and thereby reduce the solution time significantly. When a simplex optimizer starts from an existing solution it is said to perform a *warm-start*. If the user is solving a sequence of optimization problems by solving the problem, making modifications, and solving again, **MOSEK** will warm-start automatically.

By default **MOSEK** uses presolve when performing a warm-start. If the optimizer only needs very few iterations to find the optimal solution it may be better to turn off the presolve.

Numerical Difficulties in the Simplex Optimizers

Though **MOSEK** is designed to minimize numerical instability, completely avoiding it is impossible when working in finite precision. **MOSEK** treats a "numerically unexpected behavior" event inside the optimizer as a *set-back*. The user can define how many set-backs the optimizer accepts; if that number is exceeded, the optimization will be aborted. Set-backs are a way to escape long sequences where the optimizer tries to recover from an unstable situation.

Examples of set-backs are: repeated singularities when factorizing the basis matrix, repeated loss of feasibility, degeneracy problems (no progress in objective) and other events indicating numerical difficulties. If the simplex optimizer encounters a lot of set-backs the problem is usually badly scaled; in such a situation try to reformulate it into a better scaled problem. Then, if a lot of set-backs still occur, trying one or more of the following suggestions may be worthwhile:

- Raise tolerances for allowed primal or dual feasibility: increase the value of

  - *MSK_DPAR_BASIS_TOL_X* , and

  - *MSK_DPAR_BASIS_TOL_S* .

- Raise or lower pivot tolerance: Change the *MSK_DPAR_SIMPLEX_ABS_TOL_PIV* parameter.

- Switch optimizer: Try another optimizer.

- Switch off crash: Set both *MSK_IPAR_SIM_PRIMAL_CRASH* and *MSK_IPAR_SIM_DUAL_CRASH* to 0.

- Experiment with other pricing strategies: Try different values for the parameters

  - *MSK_IPAR_SIM_PRIMAL_SELECTION* and

  - *MSK_IPAR_SIM_DUAL_SELECTION* .

- If you are using warm-starts, in rare cases switching off this feature may improve stability. This is controlled by the *MSK_IPAR_SIM_HOTSTART* parameter.

- Increase maximum number of set-backs allowed controlled by *MSK_IPAR_SIM_MAX_NUM_SETBACKS* .

- If the problem repeatedly becomes infeasible try switching off the special degeneracy handling. See the parameter *MSK_IPAR_SIM_DEGEN* for details.

The Simplex Log

Below is a typical log output from the simplex optimizer:

```
Optimizer  - solved problem        : the primal
Optimizer  - Constraints           : 667
Optimizer  - Scalar variables      : 1424              conic                  : 0
Optimizer  - hotstart              : no
ITER      DEGITER(%)  PFEAS       DFEAS       POBJ              DOBJ              TIME␣
↪     TOTTIME
0         0.00        1.43e+05    NA          6.5584140832e+03  NA                0.00␣
↪     0.02
1000      1.10        0.00e+00    NA          1.4588289726e+04  NA                0.13␣
↪     0.14
2000      0.75        0.00e+00    NA          7.3705564855e+03  NA                0.21␣
↪     0.22
3000      0.67        0.00e+00    NA          6.0509727712e+03  NA                0.29␣
↪     0.31
4000      0.52        0.00e+00    NA          5.5771203906e+03  NA                0.38␣
↪     0.39
4533      0.49        0.00e+00    NA          5.5018458883e+03  NA                0.42␣
↪     0.44
```

The first lines summarize the problem the optimizer is solving. This is followed by the iteration log, with the following meaning:

- `ITER`: Number of iterations.

- `DEGITER(%)`: Ratio of degenerate iterations.

- `PFEAS`: Primal feasibility measure reported by the simplex optimizer. The numbers should be 0 if the problem is primal feasible (when the primal variant is used).

- `DFEAS`: Dual feasibility measure reported by the simplex optimizer. The number should be 0 if the problem is dual feasible (when the dual variant is used).

- `POBJ`: An estimate for the primal objective value (when the primal variant is used).

- `DOBJ`: An estimate for the dual objective value (when the dual variant is used).

- `TIME`: Time spent since this instance of the simplex optimizer was invoked (in seconds).

- `TOTTIME`: Time spent since optimization started (in seconds).

10.4 Conic Optimization

For conic optimization problems only an interior-point type optimizer is available.

10.4.1 The Interior-point optimizer

The homogeneous primal-dual problem

The interior-point optimizer is an implementation of the so-called homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see *[ART03]*. In order to keep our discussion simple we will assume that **MOSEK** solves a conic optimization problem of the form:

$$
\begin{array}{lll}
\text{minimize} & c^T x \\
\text{subject to} & Ax & = & b, \\
& x \in \mathcal{K}
\end{array}
\tag{10.6}
$$

where K is a convex cone. The corresponding dual problem is

$$
\begin{array}{lll}
\text{maximize} & b^T y \\
\text{subject to} & A^T y + s & = & c, \\
& x \in \mathcal{K}^*
\end{array}
\tag{10.7}
$$

where $\mathcal{K}^*$ is the dual cone of $\mathcal{K}$. See Section *9.2* for definitions.

Since it is not known beforehand whether problem (10.6) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason that **MOSEK** solves the so-called homogeneous model

$$
\begin{array}{rcl}
Ax - b\tau & = & 0, \\
A^T y + s - c\tau & = & 0, \\
-c^T x + b^T y - \kappa & = & 0, \\
x & \in & \mathcal{K}, \\
s & \in & \mathcal{K}^*, \\
\tau, \kappa & \geq & 0,
\end{array}
\tag{10.8}
$$

where y and s correspond to the dual variables in (10.6), and τ and κ are two additional scalar variables. Note that the homogeneous model (10.8) always has a solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (10.8) satisfies

$$(x^*)^T s^* + \tau^* \kappa^* = 0$$

i.e. complementarity. Observe that $x^* \in \mathcal{K}$ and $s^* \in \mathcal{K}^*$ implies

$$(x^*)^T s^* \geq 0$$

and therefore

$$\tau^* \kappa^* = 0.$$

since $\tau^*, \kappa^* \geq 0$. Hence, at least one of τ^* and κ^* is zero.

First, assume that $\tau^* > 0$ and hence $\kappa^* = 0$. It follows that

$$
\begin{aligned}
A\tfrac{x^*}{\tau^*} &= b, \\
A^T \tfrac{y^*}{\tau^*} + \tfrac{s^*}{\tau^*} &= c, \\
-c^T \tfrac{x^*}{\tau^*} + b^T \tfrac{y^*}{\tau^*} &= 0, \\
x^*/\tau^* &\in \mathcal{K}, \\
s^*/\tau^* &\in \mathcal{K}^*.
\end{aligned}
$$

This shows that $\frac{x^*}{\tau^*}$ is a primal optimal solution and $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$ is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left(\frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*} \right)$$

is a primal-dual optimal solution.

On other hand, if $\kappa^* > 0$ then

$$
\begin{aligned}
Ax^* &= 0, \\
A^T y^* + s^* &= 0, \\
-c^T x^* + b^T y^* &= \kappa^*, \\
x^* &\in \mathcal{K}, \\
s^* &\in \mathcal{K}^*.
\end{aligned}
$$

This implies that at least one of

$$c^T x^* < 0 \tag{10.9}$$

or

$$b^T y^* > 0 \tag{10.10}$$

holds. If (10.9) is satisfied, then x^* is a certificate of dual infeasibility, whereas if (10.10) holds then y^* is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm *[And09]*.

Interior-point Termination Criterion

Since computations are performed in finite precision, and for efficiency reasons, it is not possible to solve the homogeneous model exactly in general. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In every iteration k of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to the homogeneous model is generated, where

$$x^k \in \mathcal{K}, s^k \in \mathcal{K}^*, \tau^k, \kappa^k > 0.$$

Therefore, it is possible to compute the values:

$$
\begin{array}{rcl}
\rho_p^k & = & \arg\min_\rho \left\{ \rho \mid \left\| A\frac{x^k}{\tau^k} - b \right\|_\infty \leq \rho\varepsilon_p(1 + \|b\|_\infty) \right\}, \\
\rho_d^k & = & \arg\min_\rho \left\{ \rho \mid \left\| A^T\frac{y^k}{\tau^k} + \frac{s^k}{\tau^k} - c \right\|_\infty \leq \rho\varepsilon_d(1 + \|c\|_\infty) \right\}, \\
\rho_g^k & = & \arg\min_\rho \left\{ \rho \mid \left(\frac{(x^k)^T s^k}{(\tau^k)^2}, \left| \frac{c^T x^k}{\tau^k} - \frac{b^T y^k}{\tau^k} \right| \right) \leq \rho\varepsilon_g \max\left(1, \frac{\min\left(|c^T x^k|, |b^T y^k|\right)}{\tau^k} \right) \right\}, \\
\rho_{pi}^k & = & \arg\min_\rho \left\{ \rho \mid \left\| A^T y^k + s^k \right\|_\infty \leq \rho\varepsilon_i b^T y^k, \ b^T y^k > 0 \right\} \text{ and} \\
\rho_{di}^k & = & \arg\min_\rho \left\{ \rho \mid \left\| Ax^k \right\|_\infty \leq -\rho\varepsilon_i c^T x^k, \ c^T x^k < 0 \right\}.
\end{array}
$$

Note $\varepsilon_p, \varepsilon_d, \varepsilon_g$ and ε_i are nonnegative user specified tolerances.

Optimal Case

Observe ρ_p^k measures how far x^k/τ^k is from being a good approximate primal feasible solution. Indeed if $\rho_p^k \leq 1$, then

$$\left\| A\frac{x^k}{\tau^k} - b \right\|_\infty \leq \varepsilon_p(1 + \|b\|_\infty). \tag{10.11}$$

This shows the violations in the primal equality constraints for the solution x^k/τ^k is small compared to the size of b given ε_p is small.

Similarly, if $\rho_d^k \leq 1$, then $(y^k, s^k)/\tau^k$ is an approximate dual feasible solution. If in addition $\rho_g \leq 1$, then the solution $(x^k, y^k, s^k)/\tau^k$ is approximate optimal because the associated primal and dual objective values are almost identical.

In other words if $\max(\rho_p^k, \rho_d^k, \rho_g^k) \leq 1$, then

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is an approximate optimal solution.

Dual Infeasibility Certificate

Next assume that $\rho_{di}^k \leq 1$ and hence

$$\left\| Ax^k \right\|_\infty \leq -\varepsilon_i c^T x^k \text{ and } -c^T x^k > 0$$

holds. Now in this case the problem is declared dual infeasible and x^k is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{x} := \frac{x^k}{-c^T x^k}$$

and it is easy to verify that

$$\|A\bar{x}\|_\infty \leq \varepsilon_i \text{ and } c^T \bar{x} = -1$$

which shows $\bar{x}$ is an approximate certificate of dual infeasibility, where ε_i controls the quality of the approximation.

Primal Infeasiblity Certificate

Next assume that $\rho_{pi}^k \leq 1$ and hence

$$\left\| A^T y^k + s^k \right\|_\infty \leq \varepsilon_i b^T y^k \text{ and } b^T y^k > 0$$

holds. Now in this case the problem is declared primal infeasible and (y^k, s^k) is reported as a certificate of primal infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{y} := \frac{y^k}{b^T y^k} \text{ and } \bar{s} := \frac{s^k}{b^T y^k}$$

and it is easy to verify that

$$\left\| A^T \bar{y} + \bar{s} \right\|_\infty \leq \varepsilon_i \text{ and } b^T \bar{y} = 1$$

which shows (y^k, s^k) is an approximate certificate of dual infeasibility, where ε_i controls the quality of the approximation.

Adjusting optimality criteria and near optimality

It is possible to adjust the tolerances ε_p, ε_d, ε_g and ε_i using parameters; see table for details.

Table 10.2: Parameters employed in termination criterion

ToleranceParameter	name
ε_p	`MSK_DPAR_INTPNT_TOL_PFEAS`
ε_d	`MSK_DPAR_INTPNT_TOL_DFEAS`
ε_g	`MSK_DPAR_INTPNT_TOL_REL_GAP`
ε_i	`MSK_DPAR_INTPNT_TOL_INFEAS`

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (10.11) reveals that the quality of the solution depends on $\|b\|_\infty$ and $\|c\|_\infty$; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate *[And09]*. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances, ε_p, ε_d, ε_g and ε_i, have to be relaxed together to achieve an effect.

In some cases the interior-point method terminates having found a solution not too far from meeting the optimality condition (10.11). A solution is defined as *near optimal* if scaling the termination tolerances ε_p, ε_d, ε_g and ε_g by the same factor $\varepsilon_n \in [1.0, +\infty]$ makes the condition (10.11) satisfied. A near optimal solution is therefore of lower quality but still potentially valuable. If for instance the solver stalls, i.e. it can make no more significant progress towards the optimal solution, a near optimal solution could be available and be good enough for the user. Near infeasibility certificates are defined similarly. The value of ε_n can be adjusted with the parameter `MSK_DPAR_INTPNT_CO_TOL_NEAR_REL`.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer  - threads              : 20
Optimizer  - solved problem       : the primal
Optimizer  - Constraints          : 1
Optimizer  - Cones                : 2
```

```
Optimizer   - Scalar variables       : 6            conic                : 6
Optimizer   - Semi-definite variables: 0            scalarized           : 0
Factor      - setup time             : 0.00         dense det. time      : 0.00
Factor      - ML order time          : 0.00         GP order time        : 0.00
Factor      - nonzeros before factor : 1            after factor         : 1
Factor      - dense dim.             : 0            flops                : 1.70e+01
ITE PFEAS    DFEAS    GFEAS    PRSTATUS   POBJ              DOBJ              MU       TIME
0   1.0e+00  2.9e-01  3.4e+00  0.00e+00   2.414213562e+00   0.000000000e+00   1.0e+00  0.01
1   2.7e-01  7.9e-02  2.2e+00  8.83e-01   6.969257574e-01   -9.685901771e-03  2.7e-01  0.01
2   6.5e-02  1.9e-02  1.2e+00  1.16e+00   7.606090061e-01   6.046141322e-01   6.5e-02  0.01
3   1.7e-03  5.0e-04  2.2e-01  1.12e+00   7.084385672e-01   7.045122560e-01   1.7e-03  0.01
4   1.4e-08  4.2e-09  4.9e-08  1.00e+00   7.071067941e-01   7.071067599e-01   1.4e-08  0.01
```

The first line displays the number of threads used by the optimizer and the second line tells that the optimizer chose to solve the dual problem rather than the primal problem. The next line displays the problem dimensions as seen by the optimizer, and the `Factor...` lines show various statistics. This is followed by the iteration log.

Using the same notation as in Section *10.4.1* the columns of the iteration log have the following meaning:

- `ITE`: Iteration index k.

- `PFEAS`: $\left\| Ax^k - b\tau^k \right\|_\infty$. The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.

- `DFEAS`: $\left\| A^T y^k + s^k - c\tau^k \right\|_\infty$. The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.

- `GFEAS`: $\left| -c^T x^k + b^T y^k - \kappa^k \right|$. The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.

- `PRSTATUS`: This number converges to 1 if the problem has an optimal solution whereas it converges to -1 if that is not the case.

- `POBJ`: $c^T x^k / \tau^k$. An estimate for the primal objective value.

- `DOBJ`: $b^T y^k / \tau^k$. An estimate for the dual objective value.

- `MU`: $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$. The numbers in this column should always converge to zero.

- `TIME`: Time spent since the optimization started (in seconds).

10.5 Nonlinear Convex Optimization

10.5.1 The Interior-point Optimizer

For quadratic, quadratically constrained, and general convex optimization problems an interior-point type optimizer is available. The interior-point optimizer is an implementation of the homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see *[AY98]*, *[AY99]*.

The Convexity Requirement

Continuous nonlinear problems are required to be convex. For quadratic problems **MOSEK** test this requirement before optimizing. Specifying a non-convex problem results in an error message.

The following parameters are available to control the convexity check:

- *MSK_IPAR_CHECK_CONVEXITY* : Turn convexity check on/off.

- *MSK_DPAR_CHECK_CONVEXITY_REL_TOL* : Tolerance for convexity check.

- *MSK_IPAR_LOG_CHECK_CONVEXITY* : Turn on more log information for debugging.

The Differentiabilty Requirement

The nonlinear optimizer in **MOSEK** requires both first order and second order derivatives. This of course implies care should be taken when solving problems involving non-differentiable functions.

For instance, the function

$$f(x) = x^2$$

is differentiable everywhere whereas the function

$$f(x) = \sqrt{x}$$

is only differentiable for $x > 0$. In order to make sure that **MOSEK** evaluates the functions at points where they are differentiable, the function domains must be defined by setting appropriate variable bounds.

In general, if a variable is not ranged **MOSEK** will only evaluate that variable at points strictly within the bounds. Hence, imposing the bound

$$x \geq 0$$

in the case of $\sqrt{x}$ is sufficient to guarantee that the function will only be evaluated in points where it is differentiable.

However, if a function is differentiable on a closed range, specifying the variable bounds is not sufficient. Consider the function

$$f(x) = \frac{1}{x} + \frac{1}{1-x}. \tag{10.12}$$

In this case the bounds

$$0 \leq x \leq 1$$

will not guarantee that **MOSEK** only evaluates the function for x between 0 and 1 . To force **MOSEK** to strictly satisfy both bounds on ranged variables set the parameter `MSK_IPAR_INTPNT_STARTING_POINT` to `"MSK_STARTING_POINT_SATISFY_BOUNDS"` .

For efficiency reasons it may be better to reformulate the problem than to force **MOSEK** to observe ranged bounds strictly. For instance, (10.12) can be reformulated as follows

$$
\begin{aligned}
f(x) &= \frac{1}{x} + \frac{1}{y} \\
0 &= 1 - x - y \\
0 &\leq x \\
0 &\leq y.
\end{aligned}
$$

Interior-point Termination Criteria

The parameters controlling when the general convex interior-point optimizer terminates are shown in Table 10.3.

Table 10.3: Parameters employed in termination criteria.

Parameter name	Purpose
`MSK_DPAR_INTPNT_NL_TOL_PFEAS`	Controls primal feasibility
`MSK_DPAR_INTPNT_NL_TOL_DFEAS`	Controls dual feasibility
`MSK_DPAR_INTPNT_NL_TOL_REL_GAP`	Controls relative gap
`MSK_DPAR_INTPNT_TOL_INFEAS`	Controls when the problem is declared infeasible
`MSK_DPAR_INTPNT_NL_TOL_MU_RED`	Controls when the complementarity is reduced enough

THE OPTIMIZER FOR MIXED-INTEGER PROBLEMS

A problem is a mixed-integer optimization problem when one or more of the variables are constrained to be integer valued. Readers unfamiliar with integer optimization are recommended to consult some relevant literature, e.g. the book *[Wol98]* by Wolsey.

11.1 The Mixed-integer Optimizer Overview

MOSEK can solve mixed-integer

- linear,
- quadratic and quadratically constrained, and
- conic quadratic

problems, at least as long as they do not contain both quadratic objective or constraints and conic constraints at the same time. The mixed-integer optimizer is specialized for solving linear and conic optimization problems. Pure quadratic and quadratically constrained problems are automatically converted to conic form.

By default the mixed-integer optimizer is run-to-run deterministic. This means that if a problem is solved twice on the same computer with identical parameter settings and no time limit then the obtained solutions will be identical. If a time limit is set then this may not be case since the time taken to solve a problem is not deterministic. The mixed-integer optimizer is parallelized i.e. it can exploit multiple cores during the optimization.

The solution process can be split into these phases:

1. **Presolve:** See Section *10.1*.

2. **Cut generation:** Valid inequalities (cuts) are added to improve the lower bound.

3. **Heuristic:** Using heuristics the optimizer tries to guess a good feasible solution. Heuristics can be controlled by the parameter *MSK_IPAR_MIO_HEURISTIC_LEVEL* .

4. **Search:** The optimal solution is located by branching on integer variables.

11.2 Relaxations and bounds

It is important to understand that, in a worst-case scenario, the time required to solve integer optimization problems grows exponentially with the size of the problem (solving mixed-integer problems is NP-hard). For instance, a problem with n binary variables, may require time proportional to 2^n . The value of 2^n is huge even for moderate values of n.

In practice this implies that the focus should be on computing a near-optimal solution quickly rather than on locating an optimal solution. Even if the problem is only solved approximately, it is important to know how far the approximate solution is from an optimal one. In order to say something about the quality of an approximate solution the concept of *relaxation* is important.

Consider for example a mixed-integer optimization problem

$$
\begin{aligned}
z^* \quad = \quad & \text{minimize} & c^T x \\
& \text{subject to} & Ax &= b, \\
& & x &\geq 0 \\
& & x_j &\in \mathbb{Z}, & \forall j \in \mathcal{J}.
\end{aligned}
\tag{11.1}
$$

It has the continuous relaxation

$$
\begin{aligned}
\underline{z} \quad = \quad & \text{minimize} & c^T x \\
& \text{subject to} & Ax &= b, \\
& & x &\geq 0
\end{aligned}
\tag{11.2}
$$

obtained simply by ignoring the integrality restrictions. The relaxation is a continuous problem, and therefore much faster to solve to optimality with a linear (or, in the general case, conic) optimizer. We call the optimal value $\underline{z}$ the *objective bound*. The objective bound $\underline{z}$ normally increases during the solution search process when the continuous relaxation is gradually refined.

Moreover, if $\hat{x}$ is any feasible solution to (11.1) and

$$
\bar{z} := c^T \hat{x}
$$

then

$$
\underline{z} \leq z^* \leq \bar{z}.
$$

These two inequalities allow us to estimate the quality of the integer solution: it is no further away from the optimum than $\bar{z} - \underline{z}$ in terms of the objective value. Whenever a mixed-integer problem is solved **MOSEK** reports this lower bound so that the quality of the reported solution can be evaluated.

11.3 Termination Criterion

In general, it is time consuming to find an exact feasible and optimal solution to an integer optimization problem, though in many practical cases it may be possible to find a sufficiently good solution. The issue of terminating the mixed-integer optimizer is rather delicate and the user has numerous possibilities of influencing it with various parameters. The mixed-integer optimizer employs a relaxed feasibility and optimality criterion to determine when a satisfactory solution is located.

A candidate solution that is feasible for the continuous relaxation is said to be an *integer feasible solution* if the criterion

$$
\min(x_j - \lfloor x_j \rfloor, \lceil x_j \rceil - x_j) \leq \delta_1 \quad \forall j \in \mathcal{J}
$$

is satisfied, meaning that x_j is at most δ_1 from the nearest integer.

Whenever the integer optimizer locates an integer feasible solution it will check if the criterion

$$
\bar{z} - \underline{z} \leq \max(\delta_2, \delta_3 \max(10^{-10}, |\bar{z}|))
$$

is satisfied. If this is the case, the integer optimizer terminates and reports the integer feasible solution as an optimal solution. If an optimal solution cannot be located after the time specified by the parameter `MSK_DPAR_MIO_DISABLE_TERM_TIME` (in seconds), it may be advantageous to relax the termination criteria, and they become replaced with

$$
\bar{z} - \underline{z} \leq \max(\delta_4, \delta_5 \max(10^{-10}, |\bar{z}|)).
$$

Any solution satisfying those will now be reported as **near optimal** and the solver will be terminated (note that since this criterion depends on timing, the optimizer will not be run to run deterministic).

All the δ tolerances discussed above can be adjusted using suitable parameters — see Table 11.1.

Table 11.1: Tolerances for the mixed-integer optimizer.

Tolerance	Parameter name
δ_1	`MSK_DPAR_MIO_TOL_ABS_RELAX_INT`
δ_2	`MSK_DPAR_MIO_TOL_ABS_GAP`
δ_3	`MSK_DPAR_MIO_TOL_REL_GAP`
δ_4	`MSK_DPAR_MIO_NEAR_TOL_ABS_GAP`
δ_5	`MSK_DPAR_MIO_NEAR_TOL_REL_GAP`

In Table 11.2 some other common parameters affecting the integer optimizer termination criterion are shown. Please note that if the effect of a parameter is delayed, the associated termination criterion is applied only after some time, specified by the `MSK_DPAR_MIO_DISABLE_TERM_TIME` parameter.

Table 11.2: Other parameters affecting the integer optimizer termination criterion.

Parameter name	Delayed	Explanation
`MSK_IPAR_MIO_MAX_NUM_BRANCHES`	Yes	Maximum number of branches allowed.
`MSK_IPAR_MIO_MAX_NUM_RELAXS`	Yes	Maximum number of relaxations allowed.
`MSK_IPAR_MIO_MAX_NUM_SOLUTIONS`	Yes	Maximum number of feasible integer solutions allowed.

11.4 Speeding Up the Solution Process

As mentioned previously, in many cases it is not possible to find an optimal solution to an integer optimization problem in a reasonable amount of time. Some suggestions to reduce the solution time are:

- Relax the termination criterion: In case the run time is not acceptable, the first thing to do is to relax the termination criterion — see Section *11.3* for details.

- Specify a good initial solution: In many cases a good feasible solution is either known or easily computed using problem-specific knowledge. If a good feasible solution is known, it is usually worthwhile to use this as a starting point for the integer optimizer.

- Improve the formulation: A mixed-integer optimization problem may be impossible to solve in one form and quite easy in another form. However, it is beyond the scope of this manual to discuss good formulations for mixed-integer problems. For discussions on this topic see for example *[Wol98]*.

11.5 Understanding Solution Quality

To determine the quality of the solution one should check the following:

- The problem status and solution status returned by **MOSEK**, as well as constraint violations in case of suboptimal solutions.

- The *optimality gap* defined as

$$\epsilon = |(\text{objective value of feasible solution}) - (\text{objective bound})| = |\bar{z} - \underline{z}|.$$

which measures how much the located solution can deviate from the optimal solution to the problem. The optimality gap can be retrieved through the information item `"MSK_DINF_MIO_OBJ_ABS_GAP"`. Often it is more meaningful to look at the relative optimality gap normalized against the magnitude of the solution.

$$\epsilon_{\text{rel}} = \frac{|\bar{z} - \underline{z}|}{\max(10^{-10}, |\bar{z}|)}.$$

The relative optimality gap is available in `"MSK_DINF_MIO_OBJ_REL_GAP"`.

11.6 The Optimizer Log

Below is a typical log output from the mixed-integer optimizer:

```
Presolved problem: 6573 variables, 35728 constraints, 101258 non-zeros
Presolved problem: 0 general integer, 4294 binary, 2279 continuous
Clique table size: 1636
BRANCHES RELAXS    ACT_NDS  DEPTH   BEST_INT_OBJ          BEST_RELAX_OBJ        REL_GAP(%)  TIME
0        1         0        0       NA                    1.8218819866e+07      NA          1.6
0        1         0        0       1.8331557950e+07      1.8218819866e+07      0.61        3.5
0        1         0        0       1.8300507546e+07      1.8218819866e+07      0.45        4.3
Cut generation started.
0        2         0        0       1.8300507546e+07      1.8218819866e+07      0.45        5.3
Cut generation terminated. Time = 1.43
0        3         0        0       1.8286893047e+07      1.8231580587e+07      0.30        7.5
15       18        1        0       1.8286893047e+07      1.8231580587e+07      0.30        10.5
31       34        1        0       1.8286893047e+07      1.8231580587e+07      0.30        11.1
51       54        1        0       1.8286893047e+07      1.8231580587e+07      0.30        11.6
91       94        1        0       1.8286893047e+07      1.8231580587e+07      0.30        12.4
171      174       1        0       1.8286893047e+07      1.8231580587e+07      0.30        14.3
331      334       1        0       1.8286893047e+07      1.8231580587e+07      0.30        17.9


[ ... ]


Objective of best integer solution : 1.825846762609e+07
Best objective bound          : 1.823311032986e+07
Construct solution objective      : Not employed
Construct solution # roundings    : 0
User objective cut value          : 0
Number of cuts generated          : 117
  Number of Gomory cuts           : 108
  Number of CMIR cuts             : 9
Number of branches                : 4425
Number of relaxations solved      : 4410
Number of interior point iterations: 25
Number of simplex iterations      : 221131
```

The first lines contain a summary of the problem as seen by the optimizer. This is followed by the iteration log. The columns have the following meaning:

- `BRANCHES`: Number of branches generated.

- `RELAXS`: Number of relaxations solved.

- `ACT_NDS`: Number of active branch bound nodes.

- `DEPTH`: Depth of the recently solved node.

- `BEST_INT_OBJ`: The best integer objective value, $\bar{z}$.

- `BEST_RELAX_OBJ`: The best objective bound, $\underline{z}$.

- `REL_GAP(%)`: Relative optimality gap, $100\% \cdot \epsilon_{\mathrm{rel}}$

- `TIME`: Time (in seconds) from the start of optimization.

Following that a summary of the optimization process is printed.

API REFERENCE

12.1 Command Reference

The Rmosek interface is composed by a small set of functions and structures.

- *Function list*
- *Structures and data types list*

12.1.1 Functions

- *mosek*
- *mosek_version*
- *mosek_clean*
- *mosek_read*
- *mosek_write*

r = mosek(prob, opts)

> Solve an optimization problem using the **MOSEK** optimization library.

> > **Parameters**

> > > - prob [in] (problem) – a struct describing the optimization problem.
> > > - opts [in] (options) – the solver options

> > **Return r** (result) – a struct that contains the result of the optimization

ver = mosek_version()

> Retrieves a string containing the version number of the utilized **MOSEK** optimization library.

> > **Return ver** (int) – The version number.

void = mosek_clean()

> Forces the early release of any previously acquired **MOSEK** license. If you do not share a limited number of licenses among multiple users, you do not need to use this function. The acquisition of a new **MOSEK** license will automatically take place at the next call to the function *mosek* given a valid problem description, using a small amount of extra time.

> For advanced users: If you utilize the *.Call* convention directly, i.e. bypassing the mosek R-function definition, an Rf_error will result in an unclean memory space. For this reason you can also use this function to tidy up uncleaned resources in case an error occurs. Otherwise this cleaning will not happen until the next call to *mosek* or until the library is unloaded. This usage have not been documented elsewhere.

out = mosek_read(filepath, opts = list())

> Interprets a model from any standard modeling fileformat (e.g. lp, opf, mps, task, etc.), controlled

by a set of options. The result contains an optimization problem which is compliant with the input specifications of function *mosek*.

> **Parameters**
>
>> - `filepath [in]` (string) – A string describing the path to file, either absolute or relative to the working directory. The specified location will be the source of the optimization model to be read.
>>
>> - `opts [in]` (io_options) – The options could have any name, and are, in fact, often input directly as an anonymous list.
>
> **Return** `out` (result) – The resulting function output variable , returned by the interface, holds the response of the function call.

`r = mosek_write(prob, filepath, opts)`

> Outputs a model of an optimization problem in a file format (see Section *13*), controlled by a set of options. The modeling file format is selected based on the extension of the modelfile.
>
> **Parameters**
>
>> - `prob [in]` (problem) – The input variable could have any name, but should be a list object describing the optimization problem using the same fields as for the *mosek* function.
>>
>> - `filepath [in]` (string) – The input variable should be a string describing the path to modelfile. This path can either be absolute or relative to the working directory, and will overwrite any existing data on this location. The specified location will be the destination of the exported model.
>>
>> - `opts [in]` (io_options) – The options could have any name, and are, in fact, often specified directly as an anonymous list.
>
> **Return** `r` (result) – Function return information.

12.1.2 Structures and Data Types

`rescode`
> The return code type. See Section *12.4*.

`problem`
> A list object describing the optimization problem using the following fields.
>
> **Fields**
>
>> - `sense` (string) – Objective sense, e.g. `"max"` or `"min"`
>> - `c` (numeric_vector) – Objective coefficient array.
>> - `c0` (numeric) – Objective constant.
>> - `A` (matrix_sparse) – Constraint sparse matrix.
>> - `bc` (matrix) – Lower and upper constraint bounds
>> - `bx` (matrix) – Lower and upper variable bounds
>> - `cones` (matrix_list) – Conic constraints
>> - `bardim` (numeric_vector) – Semidefinite variable dimensions
>> - `barc` (list) – Semidefinite objective coefficients
>> - `barA` (list) – Semidefinite constraint coefficients
>> - `intsub` (numeric_vector) – Integer variable indexes
>> - `qobj` (list) – [*optional*] Quadratic convex optimization
>> - `scopt` (list) – [*optional*] Separable convex optimization

- `iparam` (list) – Integer parameter list
- `dparam` (list) – Double parameter list
- `sparam` (list) – String parameter list
- `sol` (solver_solutions) – Initial solution struct

options

The options could have any name, and are, in fact, often input directly as an anonymous list.

Fields

- `verbose` (numeric) – Output logging verbosity
- `usesol` (bool) – Whether to use the initial solution
- `useparam` (bool) – Whether to use the specified parameter settings
- `soldetail` (numeric) – Level of detail used to describe solutions
- `getinfo` (bool) – Whether to extract **MOSEK** information items
- `writebefore` (string) – Filepath used to export model
- `writeafter` (string) – Filepath used to export model and solution

solver_solutions

It contains informations about initial/final solutions.

> **Warning:** Fields availability depends on the type of problem/algorithm.

Fields

- `itr` (solution_info) – Interior solution
- `bas` (solution_info) – Basic solution
- `int` (solution_info) – Integer solution

solution_info

Fields

- `solsta` (string) – Solution status
- `prosta` (string) – Problem status
- `skc` (string_vector) – Linear constraint status keys
- `skx` (string_vector) – Variable bound status keys
- `skn` (string_vector) – Conic constraint status keys (not in basic solution)
- `xc` (numeric_vector) – Constraint activities
- `xx` (numeric_vector) – Variable activities
- `barx` (numeric_vector) – Semidefinite variable activities (not in basic solution)
- `slc` (numeric_vector) – Dual variable for constraint lower bounds (not in integer solution)
- `suc` (numeric_vector) – Dual variable for constraint upper bounds (not in integer solution)
- `slx` (numeric_vector) – Dual variable for variable lower bounds (not in integer solution)
- `sux` (numeric_vector) – Dual variable for variable lower bounds (not in integer solution)

- snx (numeric_vector) – Dual variable of conic constraints (not in basic or integer solution)

- bars (numeric_vector) – Dual variable of semidefinite domains (not in basic or integer solution)

- pobjval (numeric) – Primal objective value (Only available if requested by option soldetail)

- dobjval (numeric) – Dual objective value (not fo integer solution)

- pobjbound (numeric) – Best primal objective bound from relaxations (only for integer solution)

- maxinfeas (infeas_info) – Maximal solution infeasibilities

infeas_info

It contains information about the maximal solution infeasibilty for several problem items.

Note: This struct is only available if the option soldetail is specified.

Fields

- pbound (numeric) – In primal inequality constraints

- peq (numeric) – In primal equality constraints

- pcone (numeric) – In primal cone constraints

- dbound (numeric) – In dual inequality constraints

- deq (numeric) – In dual equality constraints

- dcone (numeric) – In dual cone constraints

- int (numeric) – In integer variables

io_options

It is used to specify options for input/output operations.

Fields

- verbose (numeric) – Output logging verbosity

- usesol (bool) – Whether to write an initial solution

- useparam (bool) – Whether to write all parameter settings

- getinfo (bool) – Whether to extract **MOSEK** information items

- scofile (string) – Source of operators read to scopt

- matrixformat (matrix_sparse) – The sparse format of the constraint matrix (only used by *mosek_read*).

result

It contains results for most of the Rmosek functions.

Note: Some fields are available only for specific functions.

Fields

- response (rescode) – Response from the **MOSEK** optimization library.

- code (numeric) – ID-code of response

- msg (string) – Human-readable message

- `prob` (problem) – Problem description (only available in *mosek_read*)
- `sol` (solution_info) – Available solutions (algorithm/problem dependent) (only available in *mosek*)
- `iinfo` (numeric_list) – Integer information list (Only available if requested by option `getinfo`).
- `dinfo` (numeric_list) – Double information list (Only available if requested by option `getinfo`).

12.2 Parameters grouped by topic

Analysis

- *MSK_DPAR_ANA_SOL_INFEAS_TOL*
- *MSK_IPAR_ANA_SOL_BASIS*
- *MSK_IPAR_ANA_SOL_PRINT_VIOLATED*
- *MSK_IPAR_LOG_ANA_PRO*

Basis identification

- *MSK_DPAR_SIM_LU_TOL_REL_PIV*
- *MSK_IPAR_BI_CLEAN_OPTIMIZER*
- *MSK_IPAR_BI_IGNORE_MAX_ITER*
- *MSK_IPAR_BI_IGNORE_NUM_ERROR*
- *MSK_IPAR_BI_MAX_ITERATIONS*
- *MSK_IPAR_INTPNT_BASIS*
- *MSK_IPAR_LOG_BI*
- *MSK_IPAR_LOG_BI_FREQ*

Conic interior-point method

- *MSK_DPAR_INTPNT_CO_TOL_DFEAS*
- *MSK_DPAR_INTPNT_CO_TOL_INFEAS*
- *MSK_DPAR_INTPNT_CO_TOL_MU_RED*
- *MSK_DPAR_INTPNT_CO_TOL_NEAR_REL*
- *MSK_DPAR_INTPNT_CO_TOL_PFEAS*
- *MSK_DPAR_INTPNT_CO_TOL_REL_GAP*

Data check

- *MSK_DPAR_DATA_SYM_MAT_TOL*
- *MSK_DPAR_DATA_SYM_MAT_TOL_HUGE*
- *MSK_DPAR_DATA_SYM_MAT_TOL_LARGE*

- *MSK_DPAR_DATA_TOL_AIJ*

- *MSK_DPAR_DATA_TOL_AIJ_HUGE*

- *MSK_DPAR_DATA_TOL_AIJ_LARGE*

- *MSK_DPAR_DATA_TOL_BOUND_INF*

- *MSK_DPAR_DATA_TOL_BOUND_WRN*

- *MSK_DPAR_DATA_TOL_C_HUGE*

- *MSK_DPAR_DATA_TOL_CJ_LARGE*

- *MSK_DPAR_DATA_TOL_QIJ*

- *MSK_DPAR_DATA_TOL_X*

- *MSK_DPAR_SEMIDEFINITE_TOL_APPROX*

- *MSK_IPAR_CHECK_CONVEXITY*

- *MSK_IPAR_LOG_CHECK_CONVEXITY*

Data input/output

- *MSK_IPAR_INFEAS_REPORT_AUTO*

- *MSK_IPAR_LOG_FILE*

- *MSK_IPAR_OPF_MAX_TERMS_PER_LINE*

- *MSK_IPAR_OPF_WRITE_HEADER*

- *MSK_IPAR_OPF_WRITE_HINTS*

- *MSK_IPAR_OPF_WRITE_PARAMETERS*

- *MSK_IPAR_OPF_WRITE_PROBLEM*

- *MSK_IPAR_OPF_WRITE_SOL_BAS*

- *MSK_IPAR_OPF_WRITE_SOL_ITG*

- *MSK_IPAR_OPF_WRITE_SOL_ITR*

- *MSK_IPAR_OPF_WRITE_SOLUTIONS*

- *MSK_IPAR_PARAM_READ_CASE_NAME*

- *MSK_IPAR_PARAM_READ_IGN_ERROR*

- *MSK_IPAR_READ_DATA_COMPRESSED*

- *MSK_IPAR_READ_DATA_FORMAT*

- *MSK_IPAR_READ_DEBUG*

- *MSK_IPAR_READ_KEEP_FREE_CON*

- *MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU*

- *MSK_IPAR_READ_LP_QUOTED_NAMES*

- *MSK_IPAR_READ_MPS_FORMAT*

- *MSK_IPAR_READ_MPS_WIDTH*

- *MSK_IPAR_READ_TASK_IGNORE_PARAM*

- *MSK_IPAR_SOL_READ_NAME_WIDTH*

- *MSK_IPAR_SOL_READ_WIDTH*

- *MSK_IPAR_WRITE_BAS_CONSTRAINTS*

- *MSK_IPAR_WRITE_BAS_HEAD*
- *MSK_IPAR_WRITE_BAS_VARIABLES*
- *MSK_IPAR_WRITE_DATA_COMPRESSED*
- *MSK_IPAR_WRITE_DATA_FORMAT*
- *MSK_IPAR_WRITE_DATA_PARAM*
- *MSK_IPAR_WRITE_FREE_CON*
- *MSK_IPAR_WRITE_GENERIC_NAMES*
- *MSK_IPAR_WRITE_GENERIC_NAMES_IO*
- *MSK_IPAR_WRITE_IGNORE_INCOMPATIBLE_ITEMS*
- *MSK_IPAR_WRITE_INT_CONSTRAINTS*
- *MSK_IPAR_WRITE_INT_HEAD*
- *MSK_IPAR_WRITE_INT_VARIABLES*
- *MSK_IPAR_WRITE_LP_FULL_OBJ*
- *MSK_IPAR_WRITE_LP_LINE_WIDTH*
- *MSK_IPAR_WRITE_LP_QUOTED_NAMES*
- *MSK_IPAR_WRITE_LP_STRICT_FORMAT*
- *MSK_IPAR_WRITE_LP_TERMS_PER_LINE*
- *MSK_IPAR_WRITE_MPS_FORMAT*
- *MSK_IPAR_WRITE_MPS_INT*
- *MSK_IPAR_WRITE_PRECISION*
- *MSK_IPAR_WRITE_SOL_BARVARIABLES*
- *MSK_IPAR_WRITE_SOL_CONSTRAINTS*
- *MSK_IPAR_WRITE_SOL_HEAD*
- *MSK_IPAR_WRITE_SOL_IGNORE_INVALID_NAMES*
- *MSK_IPAR_WRITE_SOL_VARIABLES*
- *MSK_IPAR_WRITE_TASK_INC_SOL*
- *MSK_IPAR_WRITE_XML_MODE*
- *MSK_SPAR_BAS_SOL_FILE_NAME*
- *MSK_SPAR_DATA_FILE_NAME*
- *MSK_SPAR_DEBUG_FILE_NAME*
- *MSK_SPAR_INT_SOL_FILE_NAME*
- *MSK_SPAR_ITR_SOL_FILE_NAME*
- *MSK_SPAR_MIO_DEBUG_STRING*
- *MSK_SPAR_PARAM_COMMENT_SIGN*
- *MSK_SPAR_PARAM_READ_FILE_NAME*
- *MSK_SPAR_PARAM_WRITE_FILE_NAME*
- *MSK_SPAR_READ_MPS_BOU_NAME*
- *MSK_SPAR_READ_MPS_OBJ_NAME*
- *MSK_SPAR_READ_MPS_RAN_NAME*

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- *MSK_SPAR_READ_MPS_RHS_NAME*
- *MSK_SPAR_SENSITIVITY_FILE_NAME*
- *MSK_SPAR_SENSITIVITY_RES_FILE_NAME*
- *MSK_SPAR_SOL_FILTER_XC_LOW*
- *MSK_SPAR_SOL_FILTER_XC_UPR*
- *MSK_SPAR_SOL_FILTER_XX_LOW*
- *MSK_SPAR_SOL_FILTER_XX_UPR*
- *MSK_SPAR_STAT_FILE_NAME*
- *MSK_SPAR_STAT_KEY*
- *MSK_SPAR_STAT_NAME*
- *MSK_SPAR_WRITE_LP_GEN_VAR_NAME*

Debugging

- *MSK_IPAR_AUTO_SORT_A_BEFORE_OPT*

Dual simplex

- *MSK_IPAR_SIM_DUAL_CRASH*
- *MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION*
- *MSK_IPAR_SIM_DUAL_SELECTION*

Infeasibility report

- *MSK_IPAR_INFEAS_GENERIC_NAMES*
- *MSK_IPAR_INFEAS_REPORT_LEVEL*
- *MSK_IPAR_LOG_INFEAS_ANA*

Interior-point method

- *MSK_DPAR_CHECK_CONVEXITY_REL_TOL*
- *MSK_DPAR_INTPNT_CO_TOL_DFEAS*
- *MSK_DPAR_INTPNT_CO_TOL_INFEAS*
- *MSK_DPAR_INTPNT_CO_TOL_MU_RED*
- *MSK_DPAR_INTPNT_CO_TOL_NEAR_REL*
- *MSK_DPAR_INTPNT_CO_TOL_PFEAS*
- *MSK_DPAR_INTPNT_CO_TOL_REL_GAP*
- *MSK_DPAR_INTPNT_NL_MERIT_BAL*
- *MSK_DPAR_INTPNT_NL_TOL_DFEAS*
- *MSK_DPAR_INTPNT_NL_TOL_MU_RED*
- *MSK_DPAR_INTPNT_NL_TOL_NEAR_REL*
- *MSK_DPAR_INTPNT_NL_TOL_PFEAS*

- *MSK_DPAR_INTPNT_NL_TOL_REL_GAP*
- *MSK_DPAR_INTPNT_NL_TOL_REL_STEP*
- *MSK_DPAR_INTPNT_QO_TOL_DFEAS*
- *MSK_DPAR_INTPNT_QO_TOL_INFEAS*
- *MSK_DPAR_INTPNT_QO_TOL_MU_RED*
- *MSK_DPAR_INTPNT_QO_TOL_NEAR_REL*
- *MSK_DPAR_INTPNT_QO_TOL_PFEAS*
- *MSK_DPAR_INTPNT_QO_TOL_REL_GAP*
- *MSK_DPAR_INTPNT_TOL_DFEAS*
- *MSK_DPAR_INTPNT_TOL_DSAFE*
- *MSK_DPAR_INTPNT_TOL_INFEAS*
- *MSK_DPAR_INTPNT_TOL_MU_RED*
- *MSK_DPAR_INTPNT_TOL_PATH*
- *MSK_DPAR_INTPNT_TOL_PFEAS*
- *MSK_DPAR_INTPNT_TOL_PSAFE*
- *MSK_DPAR_INTPNT_TOL_REL_GAP*
- *MSK_DPAR_INTPNT_TOL_REL_STEP*
- *MSK_DPAR_INTPNT_TOL_STEP_SIZE*
- *MSK_DPAR_QCQO_REFORMULATE_REL_DROP_TOL*
- *MSK_IPAR_BI_IGNORE_MAX_ITER*
- *MSK_IPAR_BI_IGNORE_NUM_ERROR*
- *MSK_IPAR_INTPNT_BASIS*
- *MSK_IPAR_INTPNT_DIFF_STEP*
- *MSK_IPAR_INTPNT_HOTSTART*
- *MSK_IPAR_INTPNT_MAX_ITERATIONS*
- *MSK_IPAR_INTPNT_MAX_NUM_COR*
- *MSK_IPAR_INTPNT_MAX_NUM_REFINEMENT_STEPS*
- *MSK_IPAR_INTPNT_OFF_COL_TRH*
- *MSK_IPAR_INTPNT_ORDER_METHOD*
- *MSK_IPAR_INTPNT_REGULARIZATION_USE*
- *MSK_IPAR_INTPNT_SCALING*
- *MSK_IPAR_INTPNT_SOLVE_FORM*
- *MSK_IPAR_INTPNT_STARTING_POINT*
- *MSK_IPAR_LOG_INTPNT*

License manager

- *MSK_IPAR_CACHE_LICENSE*
- *MSK_IPAR_LICENSE_DEBUG*
- *MSK_IPAR_LICENSE_PAUSE_TIME*

- *MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRNS*
- *MSK_IPAR_LICENSE_TRH_EXPIRY_WRN*
- *MSK_IPAR_LICENSE_WAIT*

Logging

- *MSK_IPAR_LOG*
- *MSK_IPAR_LOG_ANA_PRO*
- *MSK_IPAR_LOG_BI*
- *MSK_IPAR_LOG_BI_FREQ*
- *MSK_IPAR_LOG_CUT_SECOND_OPT*
- *MSK_IPAR_LOG_EXPAND*
- *MSK_IPAR_LOG_FEAS_REPAIR*
- *MSK_IPAR_LOG_FILE*
- *MSK_IPAR_LOG_INFEAS_ANA*
- *MSK_IPAR_LOG_INTPNT*
- *MSK_IPAR_LOG_MIO*
- *MSK_IPAR_LOG_MIO_FREQ*
- *MSK_IPAR_LOG_ORDER*
- *MSK_IPAR_LOG_PRESOLVE*
- *MSK_IPAR_LOG_RESPONSE*
- *MSK_IPAR_LOG_SENSITIVITY*
- *MSK_IPAR_LOG_SENSITIVITY_OPT*
- *MSK_IPAR_LOG_SIM*
- *MSK_IPAR_LOG_SIM_FREQ*
- *MSK_IPAR_LOG_STORAGE*

Mixed-integer optimization

- *MSK_DPAR_MIO_DISABLE_TERM_TIME*
- *MSK_DPAR_MIO_MAX_TIME*
- *MSK_DPAR_MIO_NEAR_TOL_ABS_GAP*
- *MSK_DPAR_MIO_NEAR_TOL_REL_GAP*
- *MSK_DPAR_MIO_REL_GAP_CONST*
- *MSK_DPAR_MIO_TOL_ABS_GAP*
- *MSK_DPAR_MIO_TOL_ABS_RELAX_INT*
- *MSK_DPAR_MIO_TOL_FEAS*
- *MSK_DPAR_MIO_TOL_REL_DUAL_BOUND_IMPROVEMENT*
- *MSK_DPAR_MIO_TOL_REL_GAP*
- *MSK_IPAR_LOG_MIO*

- *MSK_IPAR_LOG_MIO_FREQ*

- *MSK_IPAR_MIO_BRANCH_DIR*

- *MSK_IPAR_MIO_CONSTRUCT_SOL*

- *MSK_IPAR_MIO_CUT_CLIQUE*

- *MSK_IPAR_MIO_CUT_CMIR*

- *MSK_IPAR_MIO_CUT_GMI*

- *MSK_IPAR_MIO_CUT_IMPLIED_BOUND*

- *MSK_IPAR_MIO_CUT_KNAPSACK_COVER*

- *MSK_IPAR_MIO_CUT_SELECTION_LEVEL*

- *MSK_IPAR_MIO_HEURISTIC_LEVEL*

- *MSK_IPAR_MIO_MAX_NUM_BRANCHES*

- *MSK_IPAR_MIO_MAX_NUM_RELAXS*

- *MSK_IPAR_MIO_MAX_NUM_SOLUTIONS*

- *MSK_IPAR_MIO_NODE_OPTIMIZER*

- *MSK_IPAR_MIO_NODE_SELECTION*

- *MSK_IPAR_MIO_PERSPECTIVE_REFORMULATE*

- *MSK_IPAR_MIO_PROBING_LEVEL*

- *MSK_IPAR_MIO_RINS_MAX_NODES*

- *MSK_IPAR_MIO_ROOT_OPTIMIZER*

- *MSK_IPAR_MIO_ROOT_REPEAT_PRESOLVE_LEVEL*

- *MSK_IPAR_MIO_VB_DETECTION_LEVEL*

Nonlinear convex method

- *MSK_DPAR_INTPNT_NL_MERIT_BAL*

- *MSK_DPAR_INTPNT_NL_TOL_DFEAS*

- *MSK_DPAR_INTPNT_NL_TOL_MU_RED*

- *MSK_DPAR_INTPNT_NL_TOL_NEAR_REL*

- *MSK_DPAR_INTPNT_NL_TOL_PFEAS*

- *MSK_DPAR_INTPNT_NL_TOL_REL_GAP*

- *MSK_DPAR_INTPNT_NL_TOL_REL_STEP*

- *MSK_DPAR_INTPNT_TOL_INFEAS*

- *MSK_IPAR_CHECK_CONVEXITY*

- *MSK_IPAR_LOG_CHECK_CONVEXITY*

Output information

- *MSK_IPAR_INFEAS_REPORT_LEVEL*

- *MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRNS*

- *MSK_IPAR_LICENSE_TRH_EXPIRY_WRN*

- *MSK_IPAR_LOG*

- *MSK_IPAR_LOG_BI*

- *MSK_IPAR_LOG_BI_FREQ*

- *MSK_IPAR_LOG_CUT_SECOND_OPT*

- *MSK_IPAR_LOG_EXPAND*

- *MSK_IPAR_LOG_FEAS_REPAIR*

- *MSK_IPAR_LOG_FILE*

- *MSK_IPAR_LOG_INFEAS_ANA*

- *MSK_IPAR_LOG_INTPNT*

- *MSK_IPAR_LOG_MIO*

- *MSK_IPAR_LOG_MIO_FREQ*

- *MSK_IPAR_LOG_ORDER*

- *MSK_IPAR_LOG_RESPONSE*

- *MSK_IPAR_LOG_SENSITIVITY*

- *MSK_IPAR_LOG_SENSITIVITY_OPT*

- *MSK_IPAR_LOG_SIM*

- *MSK_IPAR_LOG_SIM_FREQ*

- *MSK_IPAR_LOG_SIM_MINOR*

- *MSK_IPAR_LOG_STORAGE*

- *MSK_IPAR_MAX_NUM_WARNINGS*

Overall solver

- *MSK_IPAR_BI_CLEAN_OPTIMIZER*

- *MSK_IPAR_INFEAS_PREFER_PRIMAL*

- *MSK_IPAR_LICENSE_WAIT*

- *MSK_IPAR_MIO_MODE*

- *MSK_IPAR_OPTIMIZER*

- *MSK_IPAR_PRESOLVE_LEVEL*

- *MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS*

- *MSK_IPAR_PRESOLVE_USE*

- *MSK_IPAR_PRIMAL_REPAIR_OPTIMIZER*

- *MSK_IPAR_SENSITIVITY_ALL*

- *MSK_IPAR_SENSITIVITY_OPTIMIZER*

- *MSK_IPAR_SENSITIVITY_TYPE*

- *MSK_IPAR_SOLUTION_CALLBACK*

Overall system

- *MSK_IPAR_AUTO_UPDATE_SOL_INFO*
- *MSK_IPAR_INTPNT_MULTI_THREAD*
- *MSK_IPAR_LICENSE_WAIT*
- *MSK_IPAR_LOG_STORAGE*
- *MSK_IPAR_MIO_MT_USER_CB*
- *MSK_IPAR_MT_SPINCOUNT*
- *MSK_IPAR_NUM_THREADS*
- *MSK_IPAR_REMOVE_UNUSED_SOLUTIONS*
- *MSK_IPAR_TIMING_LEVEL*
- *MSK_SPAR_REMOTE_ACCESS_TOKEN*

Presolve

- *MSK_DPAR_PRESOLVE_TOL_ABS_LINDEP*
- *MSK_DPAR_PRESOLVE_TOL_AIJ*
- *MSK_DPAR_PRESOLVE_TOL_REL_LINDEP*
- *MSK_DPAR_PRESOLVE_TOL_S*
- *MSK_DPAR_PRESOLVE_TOL_X*
- *MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL*
- *MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_NUM_TRIES*
- *MSK_IPAR_PRESOLVE_LEVEL*
- *MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH*
- *MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH*
- *MSK_IPAR_PRESOLVE_LINDEP_USE*
- *MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS*
- *MSK_IPAR_PRESOLVE_USE*

Primal simplex

- *MSK_IPAR_SIM_PRIMAL_CRASH*
- *MSK_IPAR_SIM_PRIMAL_RESTRICT_SELECTION*
- *MSK_IPAR_SIM_PRIMAL_SELECTION*

Progress callback

- *MSK_IPAR_SOLUTION_CALLBACK*

Simplex optimizer

- *MSK_DPAR_BASIS_REL_TOL_S*
- *MSK_DPAR_BASIS_TOL_S*
- *MSK_DPAR_BASIS_TOL_X*
- *MSK_DPAR_SIM_LU_TOL_REL_PIV*
- *MSK_DPAR_SIMPLEX_ABS_TOL_PIV*
- *MSK_IPAR_BASIS_SOLVE_USE_PLUS_ONE*
- *MSK_IPAR_LOG_SIM*
- *MSK_IPAR_LOG_SIM_FREQ*
- *MSK_IPAR_LOG_SIM_MINOR*
- *MSK_IPAR_SENSITIVITY_OPTIMIZER*
- *MSK_IPAR_SIM_BASIS_FACTOR_USE*
- *MSK_IPAR_SIM_DEGEN*
- *MSK_IPAR_SIM_DUAL_PHASEONE_METHOD*
- *MSK_IPAR_SIM_EXPLOIT_DUPVEC*
- *MSK_IPAR_SIM_HOTSTART*
- *MSK_IPAR_SIM_HOTSTART_LU*
- *MSK_IPAR_SIM_MAX_ITERATIONS*
- *MSK_IPAR_SIM_MAX_NUM_SETBACKS*
- *MSK_IPAR_SIM_NON_SINGULAR*
- *MSK_IPAR_SIM_PRIMAL_PHASEONE_METHOD*
- *MSK_IPAR_SIM_REFACTOR_FREQ*
- *MSK_IPAR_SIM_REFORMULATION*
- *MSK_IPAR_SIM_SAVE_LU*
- *MSK_IPAR_SIM_SCALING*
- *MSK_IPAR_SIM_SCALING_METHOD*
- *MSK_IPAR_SIM_SOLVE_FORM*
- *MSK_IPAR_SIM_STABILITY_PRIORITY*
- *MSK_IPAR_SIM_SWITCH_OPTIMIZER*

Solution input/output

- *MSK_IPAR_INFEAS_REPORT_AUTO*
- *MSK_IPAR_SOL_FILTER_KEEP_BASIC*
- *MSK_IPAR_SOL_FILTER_KEEP_RANGED*
- *MSK_IPAR_SOL_READ_NAME_WIDTH*
- *MSK_IPAR_SOL_READ_WIDTH*
- *MSK_IPAR_WRITE_BAS_CONSTRAINTS*
- *MSK_IPAR_WRITE_BAS_HEAD*

- *MSK_IPAR_WRITE_BAS_VARIABLES*

- *MSK_IPAR_WRITE_INT_CONSTRAINTS*

- *MSK_IPAR_WRITE_INT_HEAD*

- *MSK_IPAR_WRITE_INT_VARIABLES*

- *MSK_IPAR_WRITE_SOL_BARVARIABLES*

- *MSK_IPAR_WRITE_SOL_CONSTRAINTS*

- *MSK_IPAR_WRITE_SOL_HEAD*

- *MSK_IPAR_WRITE_SOL_IGNORE_INVALID_NAMES*

- *MSK_IPAR_WRITE_SOL_VARIABLES*

- *MSK_SPAR_BAS_SOL_FILE_NAME*

- *MSK_SPAR_INT_SOL_FILE_NAME*

- *MSK_SPAR_ITR_SOL_FILE_NAME*

- *MSK_SPAR_SOL_FILTER_XC_LOW*

- *MSK_SPAR_SOL_FILTER_XC_UPR*

- *MSK_SPAR_SOL_FILTER_XX_LOW*

- *MSK_SPAR_SOL_FILTER_XX_UPR*

Termination criteria

- *MSK_DPAR_BASIS_REL_TOL_S*

- *MSK_DPAR_BASIS_TOL_S*

- *MSK_DPAR_BASIS_TOL_X*

- *MSK_DPAR_INTPNT_CO_TOL_DFEAS*

- *MSK_DPAR_INTPNT_CO_TOL_INFEAS*

- *MSK_DPAR_INTPNT_CO_TOL_MU_RED*

- *MSK_DPAR_INTPNT_CO_TOL_NEAR_REL*

- *MSK_DPAR_INTPNT_CO_TOL_PFEAS*

- *MSK_DPAR_INTPNT_CO_TOL_REL_GAP*

- *MSK_DPAR_INTPNT_NL_TOL_DFEAS*

- *MSK_DPAR_INTPNT_NL_TOL_MU_RED*

- *MSK_DPAR_INTPNT_NL_TOL_NEAR_REL*

- *MSK_DPAR_INTPNT_NL_TOL_PFEAS*

- *MSK_DPAR_INTPNT_NL_TOL_REL_GAP*

- *MSK_DPAR_INTPNT_QO_TOL_DFEAS*

- *MSK_DPAR_INTPNT_QO_TOL_INFEAS*

- *MSK_DPAR_INTPNT_QO_TOL_MU_RED*

- *MSK_DPAR_INTPNT_QO_TOL_NEAR_REL*

- *MSK_DPAR_INTPNT_QO_TOL_PFEAS*

- *MSK_DPAR_INTPNT_QO_TOL_REL_GAP*

- *MSK_DPAR_INTPNT_TOL_DFEAS*

- *MSK_DPAR_INTPNT_TOL_INFEAS*
- *MSK_DPAR_INTPNT_TOL_MU_RED*
- *MSK_DPAR_INTPNT_TOL_PFEAS*
- *MSK_DPAR_INTPNT_TOL_REL_GAP*
- *MSK_DPAR_LOWER_OBJ_CUT*
- *MSK_DPAR_LOWER_OBJ_CUT_FINITE_TRH*
- *MSK_DPAR_MIO_DISABLE_TERM_TIME*
- *MSK_DPAR_MIO_MAX_TIME*
- *MSK_DPAR_MIO_NEAR_TOL_REL_GAP*
- *MSK_DPAR_MIO_REL_GAP_CONST*
- *MSK_DPAR_MIO_TOL_REL_GAP*
- *MSK_DPAR_OPTIMIZER_MAX_TIME*
- *MSK_DPAR_UPPER_OBJ_CUT*
- *MSK_DPAR_UPPER_OBJ_CUT_FINITE_TRH*
- *MSK_IPAR_BI_MAX_ITERATIONS*
- *MSK_IPAR_INTPNT_MAX_ITERATIONS*
- *MSK_IPAR_MIO_MAX_NUM_BRANCHES*
- *MSK_IPAR_MIO_MAX_NUM_SOLUTIONS*
- *MSK_IPAR_SIM_MAX_ITERATIONS*

Other

- *MSK_IPAR_COMPRESS_STATFILE*

12.3 Parameters (alphabetical list sorted by type)

- *Double parameters*
- *Integer parameters*
- *String parameters*

12.3.1 Double parameters

dparam
> The enumeration type containing all double parameters.

MSK_DPAR_ANA_SOL_INFEAS_TOL
> If a constraint violates its bound with an amount larger than this value, the constraint name, index and violation will be printed by the solution analyzer.
>
>> **Default** 1e-6
>>
>> **Accepted** [0.0; +inf]
>>
>> **Groups** *Analysis*

MSK_DPAR_BASIS_REL_TOL_S
> Maximum relative dual bound violation allowed in an optimal basic solution.

Default 1.0e-12

Accepted [0.0; +inf]

Groups *Simplex optimizer*, *Termination criteria*

MSK_DPAR_BASIS_TOL_S
Maximum absolute dual bound violation in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Groups *Simplex optimizer*, *Termination criteria*

MSK_DPAR_BASIS_TOL_X
Maximum absolute primal bound violation allowed in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Groups *Simplex optimizer*, *Termination criteria*

MSK_DPAR_CHECK_CONVEXITY_REL_TOL
This parameter controls when the full convexity check declares a problem to be non-convex. Increasing this tolerance relaxes the criteria for declaring the problem non-convex.

A problem is declared non-convex if negative (positive) pivot elements are detected in the Cholesky factor of a matrix which is required to be PSD (NSD). This parameter controls how much this non-negativity requirement may be violated.

If d_i is the pivot element for column i, then the matrix Q is considered to not be PSD if:

$$d_i \leq -|Q_{ii}|\texttt{check\_convexity\_rel\_tol}$$

Default 1e-10

Accepted [0; +inf]

Groups *Interior-point method*

MSK_DPAR_DATA_SYM_MAT_TOL
Absolute zero tolerance for elements in in suymmetric matrixes. If any value in a symmetric matrix is smaller than this parameter in absolute terms **MOSEK** will treat the values as zero and generate a warning.

Default 1.0e-12

Accepted [1.0e-16; 1.0e-6]

Groups *Data check*

MSK_DPAR_DATA_SYM_MAT_TOL_HUGE
An element in a symmetric matrix which is larger than this value in absolute size causes an error.

Default 1.0e20

Accepted [0.0; +inf]

Groups *Data check*

MSK_DPAR_DATA_SYM_MAT_TOL_LARGE
An element in a symmetric matrix which is larger than this value in absolute size causes a warning message to be printed.

Default 1.0e10

Accepted [0.0; +inf]

Groups *Data check*

`MSK_DPAR_DATA_TOL_AIJ`

Absolute zero tolerance for elements in A. If any value A_{ij} is smaller than this parameter in absolute terms **MOSEK** will treat the values as zero and generate a warning.

Default 1.0e-12

Accepted [1.0e-16; 1.0e-6]

Groups *Data check*

`MSK_DPAR_DATA_TOL_AIJ_HUGE`

An element in A which is larger than this value in absolute size causes an error.

Default 1.0e20

Accepted [0.0; +inf]

Groups *Data check*

`MSK_DPAR_DATA_TOL_AIJ_LARGE`

An element in A which is larger than this value in absolute size causes a warning message to be printed.

Default 1.0e10

Accepted [0.0; +inf]

Groups *Data check*

`MSK_DPAR_DATA_TOL_BOUND_INF`

Any bound which in absolute value is greater than this parameter is considered infinite.

Default 1.0e16

Accepted [0.0; +inf]

Groups *Data check*

`MSK_DPAR_DATA_TOL_BOUND_WRN`

If a bound value is larger than this value in absolute size, then a warning message is issued.

Default 1.0e8

Accepted [0.0; +inf]

Groups *Data check*

`MSK_DPAR_DATA_TOL_C_HUGE`

An element in c which is larger than the value of this parameter in absolute terms is considered to be huge and generates an error.

Default 1.0e16

Accepted [0.0; +inf]

Groups *Data check*

`MSK_DPAR_DATA_TOL_CJ_LARGE`

An element in c which is larger than this value in absolute terms causes a warning message to be printed.

Default 1.0e8

Accepted [0.0; +inf]

Groups *Data check*

`MSK_DPAR_DATA_TOL_QIJ`

Absolute zero tolerance for elements in Q matrices.

Default 1.0e-16

Accepted [0.0; +inf]

> **Groups** *Data check*

MSK_DPAR_DATA_TOL_X
> Zero tolerance for constraints and variables i.e. if the distance between the lower and upper bound is less than this value, then the lower and upper bound is considered identical.
>
> > **Default** 1.0e-8
> >
> > **Accepted** [0.0; +inf]
> >
> > **Groups** *Data check*

MSK_DPAR_INTPNT_CO_TOL_DFEAS
> Dual feasibility tolerance used by the conic interior-point optimizer.
>
> > **Default** 1.0e-8
> >
> > **Accepted** [0.0; 1.0]
> >
> > **Groups** *Interior-point method*, *Termination criteria*, *Conic interior-point method*
> >
> > **See also** *MSK_DPAR_INTPNT_CO_TOL_NEAR_REL*

MSK_DPAR_INTPNT_CO_TOL_INFEAS
> Controls when the conic interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.
>
> > **Default** 1.0e-10
> >
> > **Accepted** [0.0; 1.0]
> >
> > **Groups** *Interior-point method*, *Termination criteria*, *Conic interior-point method*

MSK_DPAR_INTPNT_CO_TOL_MU_RED
> Relative complementarity gap feasibility tolerance used by the conic interior-point optimizer.
>
> > **Default** 1.0e-8
> >
> > **Accepted** [0.0; 1.0]
> >
> > **Groups** *Interior-point method*, *Termination criteria*, *Conic interior-point method*

MSK_DPAR_INTPNT_CO_TOL_NEAR_REL
> If **MOSEK** cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.
>
> > **Default** 1000
> >
> > **Accepted** [1.0; +inf]
> >
> > **Groups** *Interior-point method*, *Termination criteria*, *Conic interior-point method*

MSK_DPAR_INTPNT_CO_TOL_PFEAS
> Primal feasibility tolerance used by the conic interior-point optimizer.
>
> > **Default** 1.0e-8
> >
> > **Accepted** [0.0; 1.0]
> >
> > **Groups** *Interior-point method*, *Termination criteria*, *Conic interior-point method*
> >
> > **See also** *MSK_DPAR_INTPNT_CO_TOL_NEAR_REL*

MSK_DPAR_INTPNT_CO_TOL_REL_GAP
> Relative gap termination tolerance used by the conic interior-point optimizer.
>
> > **Default** 1.0e-7
> >
> > **Accepted** [0.0; 1.0]
> >
> > **Groups** *Interior-point method*, *Termination criteria*, *Conic interior-point method*
> >
> > **See also** *MSK_DPAR_INTPNT_CO_TOL_NEAR_REL*

MSK_DPAR_INTPNT_NL_MERIT_BAL
> Controls if the complementarity and infeasibility is converging to zero at about equal rates.
>
>> **Default** 1.0e-4
>>
>> **Accepted** [0.0; 0.99]
>>
>> **Groups** *Interior-point method*, *Nonlinear convex method*

MSK_DPAR_INTPNT_NL_TOL_DFEAS
> Dual feasibility tolerance used when a nonlinear model is solved.
>
>> **Default** 1.0e-8
>>
>> **Accepted** [0.0; 1.0]
>>
>> **Groups** *Interior-point method*, *Termination criteria*, *Nonlinear convex method*

MSK_DPAR_INTPNT_NL_TOL_MU_RED
> Relative complementarity gap tolerance.
>
>> **Default** 1.0e-12
>>
>> **Accepted** [0.0; 1.0]
>>
>> **Groups** *Interior-point method*, *Termination criteria*, *Nonlinear convex method*

MSK_DPAR_INTPNT_NL_TOL_NEAR_REL
> If the **MOSEK** nonlinear interior-point optimizer cannot compute a solution that has the pre-scribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.
>
>> **Default** 1000.0
>>
>> **Accepted** [1.0; +inf]
>>
>> **Groups** *Interior-point method*, *Termination criteria*, *Nonlinear convex method*

MSK_DPAR_INTPNT_NL_TOL_PFEAS
> Primal feasibility tolerance used when a nonlinear model is solved.
>
>> **Default** 1.0e-8
>>
>> **Accepted** [0.0; 1.0]
>>
>> **Groups** *Interior-point method*, *Termination criteria*, *Nonlinear convex method*

MSK_DPAR_INTPNT_NL_TOL_REL_GAP
> Relative gap termination tolerance for nonlinear problems.
>
>> **Default** 1.0e-6
>>
>> **Accepted** [1.0e-14; +inf]
>>
>> **Groups** *Termination criteria*, *Interior-point method*, *Nonlinear convex method*

MSK_DPAR_INTPNT_NL_TOL_REL_STEP
> Relative step size to the boundary for general nonlinear optimization problems.
>
>> **Default** 0.995
>>
>> **Accepted** [1.0e-4; 0.9999999]
>>
>> **Groups** *Interior-point method*, *Nonlinear convex method*

MSK_DPAR_INTPNT_QO_TOL_DFEAS
> Dual feasibility tolerance used when the interior-point optimizer is applied to a quadratic optimiza-tion rroblem..
>
>> **Default** 1.0e-8
>>
>> **Accepted** [0.0; 1.0]

Groups *Interior-point method*, *Termination criteria*

See also `MSK_DPAR_INTPNT_QO_TOL_NEAR_REL`

MSK_DPAR_INTPNT_QO_TOL_INFEAS

Controls when the conic interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

Default 1.0e-10

Accepted [0.0; 1.0]

Groups *Interior-point method*, *Termination criteria*

MSK_DPAR_INTPNT_QO_TOL_MU_RED

Relative complementarity gap feasibility tolerance used when interior-point optimizer is applied to a quadratic optimization problem.

Default 1.0e-8

Accepted [0.0; 1.0]

Groups *Interior-point method*, *Termination criteria*

MSK_DPAR_INTPNT_QO_TOL_NEAR_REL

If **MOSEK** cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

Default 1000

Accepted [1.0; +inf]

Groups *Interior-point method*, *Termination criteria*

MSK_DPAR_INTPNT_QO_TOL_PFEAS

Primal feasibility tolerance used when the interior-point optimizer is applied to a quadratic optimization problem.

Default 1.0e-8

Accepted [0.0; 1.0]

Groups *Interior-point method*, *Termination criteria*

See also `MSK_DPAR_INTPNT_QO_TOL_NEAR_REL`

MSK_DPAR_INTPNT_QO_TOL_REL_GAP

Relative gap termination tolerance used when the interior-point optimizer is applied to a quadratic optimization problem.

Default 1.0e-8

Accepted [0.0; 1.0]

Groups *Interior-point method*, *Termination criteria*

See also `MSK_DPAR_INTPNT_QO_TOL_NEAR_REL`

MSK_DPAR_INTPNT_TOL_DFEAS

Dual feasibility tolerance used for linear and quadratic optimization problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Groups *Interior-point method*, *Termination criteria*

MSK_DPAR_INTPNT_TOL_DSAFE

Controls the initial dual starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it might be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Groups *Interior-point method*

MSK_DPAR_INTPNT_TOL_INFEAS
    Controls when the optimizer declares the model primal or dual infeasible. A small number means
    the optimizer gets more conservative about declaring the model infeasible.

Default 1.0e-10

Accepted [0.0; 1.0]

Groups *Interior-point method*, *Termination criteria*, *Nonlinear convex method*

MSK_DPAR_INTPNT_TOL_MU_RED
    Relative complementarity gap tolerance.

Default 1.0e-16

Accepted [0.0; 1.0]

Groups *Interior-point method*, *Termination criteria*

MSK_DPAR_INTPNT_TOL_PATH
    Controls how close the interior-point optimizer follows the central path. A large value of this
    parameter means the central is followed very closely. On numerical unstable problems it may be
    worthwhile to increase this parameter.

Default 1.0e-8

Accepted [0.0; 0.9999]

Groups *Interior-point method*

MSK_DPAR_INTPNT_TOL_PFEAS
    Primal feasibility tolerance used for linear and quadratic optimization problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Groups *Interior-point method*, *Termination criteria*

MSK_DPAR_INTPNT_TOL_PSAFE
    Controls the initial primal starting point used by the interior-point optimizer. If the interior-point
    optimizer converges slowly and/or the constraint or variable bounds are very large, then it may be
    worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Groups *Interior-point method*

MSK_DPAR_INTPNT_TOL_REL_GAP
    Relative gap termination tolerance.

Default 1.0e-8

Accepted [1.0e-14; +inf]

Groups *Termination criteria*, *Interior-point method*

MSK_DPAR_INTPNT_TOL_REL_STEP
    Relative step size to the boundary for linear and quadratic optimization problems.

Default 0.9999

Accepted [1.0e-4; 0.999999]

Groups *Interior-point method*

MSK_DPAR_INTPNT_TOL_STEP_SIZE

> Minimal step size tolerance. If the step size falls below the value of this parameter, then the interior-point optimizer assumes that it is stalled. In other words the interior-point optimizer does not make any progress and therefore it is better stop.
>
> > **Default** 1.0e-6
> >
> > **Accepted** [0.0; 1.0]
> >
> > **Groups** *Interior-point method*

MSK_DPAR_LOWER_OBJ_CUT

> If either a primal or dual feasible solution is found proving that the optimal objective value is outside, the interval [*MSK_DPAR_LOWER_OBJ_CUT* , *MSK_DPAR_UPPER_OBJ_CUT*], then **MOSEK** is terminated.
>
> > **Default** -1.0e30
> >
> > **Accepted** [-inf; +inf]
> >
> > **Groups** *Termination criteria*
> >
> > **See also** *MSK_DPAR_LOWER_OBJ_CUT_FINITE_TRH*

MSK_DPAR_LOWER_OBJ_CUT_FINITE_TRH

> If the lower objective cut is less than the value of this parameter value, then the lower objective cut i.e. *MSK_DPAR_LOWER_OBJ_CUT* is treated as $-\infty$.
>
> > **Default** -0.5e30
> >
> > **Accepted** [-inf; +inf]
> >
> > **Groups** *Termination criteria*

MSK_DPAR_MIO_DISABLE_TERM_TIME

> > This parameter specifies the number of seconds n during which the termination criteria governed by
> >
> > - *MSK_IPAR_MIO_MAX_NUM_RELAXS*
> >
> > - *MSK_IPAR_MIO_MAX_NUM_BRANCHES*
> >
> > - *MSK_DPAR_MIO_NEAR_TOL_ABS_GAP*
> >
> > - *MSK_DPAR_MIO_NEAR_TOL_REL_GAP*
>
> is disabled since the beginning of the optimization.
>
> A negative value is identical to infinity i.e. the termination criteria are never checked.
>
> > **Default** -1.0
> >
> > **Accepted** [-inf; +inf]
> >
> > **Groups** *Mixed-integer optimization*, *Termination criteria*
> >
> > **See also** *MSK_IPAR_MIO_MAX_NUM_RELAXS* , *MSK_IPAR_MIO_MAX_NUM_BRANCHES* , *MSK_DPAR_MIO_NEAR_TOL_ABS_GAP* , *MSK_DPAR_MIO_NEAR_TOL_REL_GAP*

MSK_DPAR_MIO_MAX_TIME

> This parameter limits the maximum time spent by the mixed-integer optimizer. A negative number means infinity.
>
> > **Default** -1.0
> >
> > **Accepted** [-inf; +inf]
> >
> > **Groups** *Mixed-integer optimization*, *Termination criteria*

MSK_DPAR_MIO_NEAR_TOL_ABS_GAP

> Relaxed absolute optimality tolerance employed by the mixed-integer optimizer. This termination criteria is delayed. See *MSK_DPAR_MIO_DISABLE_TERM_TIME* for details.
>
> > **Default** 0.0
> >
> > **Accepted** [0.0; +inf]
> >
> > **Groups** *Mixed-integer optimization*
> >
> > **See also** *MSK_DPAR_MIO_DISABLE_TERM_TIME*

MSK_DPAR_MIO_NEAR_TOL_REL_GAP

> The mixed-integer optimizer is terminated when this tolerance is satisfied. This termination criteria is delayed. See *MSK_DPAR_MIO_DISABLE_TERM_TIME* for details.
>
> > **Default** 1.0e-3
> >
> > **Accepted** [0.0; +inf]
> >
> > **Groups** *Mixed-integer optimization*, *Termination criteria*
> >
> > **See also** *MSK_DPAR_MIO_DISABLE_TERM_TIME*

MSK_DPAR_MIO_REL_GAP_CONST

> This value is used to compute the relative gap for the solution to an integer optimization problem.
>
> > **Default** 1.0e-10
> >
> > **Accepted** [1.0e-15; +inf]
> >
> > **Groups** *Mixed-integer optimization*, *Termination criteria*

MSK_DPAR_MIO_TOL_ABS_GAP

> Absolute optimality tolerance employed by the mixed-integer optimizer.
>
> > **Default** 0.0
> >
> > **Accepted** [0.0; +inf]
> >
> > **Groups** *Mixed-integer optimization*

MSK_DPAR_MIO_TOL_ABS_RELAX_INT

> Absolute integer feasibility tolerance. If the distance to the nearest integer is less than this tolerance then an integer constraint is assumed to be satisfied.
>
> > **Default** 1.0e-5
> >
> > **Accepted** [1e-9; +inf]
> >
> > **Groups** *Mixed-integer optimization*

MSK_DPAR_MIO_TOL_FEAS

> Feasibility tolerance for mixed integer solver.
>
> > **Default** 1.0e-6
> >
> > **Accepted** [1e-9; 1e-3]
> >
> > **Groups** *Mixed-integer optimization*

MSK_DPAR_MIO_TOL_REL_DUAL_BOUND_IMPROVEMENT

> If the relative improvement of the dual bound is smaller than this value, the solver will terminate the root cut generation. A value of 0.0 means that the value is selected automatically.
>
> > **Default** 0.0
> >
> > **Accepted** [0.0; 1.0]
> >
> > **Groups** *Mixed-integer optimization*

MSK_DPAR_MIO_TOL_REL_GAP

> Relative optimality tolerance employed by the mixed-integer optimizer.

> **Default** 1.0e-4
>
> **Accepted** [0.0; +inf]
>
> **Groups** *Mixed-integer optimization*, *Termination criteria*

MSK_DPAR_OPTIMIZER_MAX_TIME
Maximum amount of time the optimizer is allowed to spent on the optimization. A negative number means infinity.

> **Default** -1.0
>
> **Accepted** [-inf; +inf]
>
> **Groups** *Termination criteria*

MSK_DPAR_PRESOLVE_TOL_ABS_LINDEP
Absolute tolerance employed by the linear dependency checker.

> **Default** 1.0e-6
>
> **Accepted** [0.0; +inf]
>
> **Groups** *Presolve*

MSK_DPAR_PRESOLVE_TOL_AIJ
Absolute zero tolerance employed for a_{ij} in the presolve.

> **Default** 1.0e-12
>
> **Accepted** [1.0e-15; +inf]
>
> **Groups** *Presolve*

MSK_DPAR_PRESOLVE_TOL_REL_LINDEP
Relative tolerance employed by the linear dependency checker.

> **Default** 1.0e-10
>
> **Accepted** [0.0; +inf]
>
> **Groups** *Presolve*

MSK_DPAR_PRESOLVE_TOL_S
Absolute zero tolerance employed for s_i in the presolve.

> **Default** 1.0e-8
>
> **Accepted** [0.0; +inf]
>
> **Groups** *Presolve*

MSK_DPAR_PRESOLVE_TOL_X
Absolute zero tolerance employed for x_j in the presolve.

> **Default** 1.0e-8
>
> **Accepted** [0.0; +inf]
>
> **Groups** *Presolve*

MSK_DPAR_QCQO_REFORMULATE_REL_DROP_TOL
This parameter determines when columns are dropped in incomplete Cholesky factorization during reformulation of quadratic problems.

> **Default** 1e-15
>
> **Accepted** [0; +inf]
>
> **Groups** *Interior-point method*

MSK_DPAR_SEMIDEFINITE_TOL_APPROX
Tolerance to define a matrix to be positive semidefinite.

Default 1.0e-10

Accepted [1.0e-15; +inf]

Groups *Data check*

MSK_DPAR_SIM_LU_TOL_REL_PIV

Relative pivot tolerance employed when computing the LU factorization of the basis in the simplex optimizers and in the basis identification procedure.

A value closer to 1.0 generally improves numerical stability but typically also implies an increase in the computational work.

Default 0.01

Accepted [1.0e-6; 0.999999]

Groups *Basis identification*, *Simplex optimizer*

MSK_DPAR_SIMPLEX_ABS_TOL_PIV

Absolute pivot tolerance employed by the simplex optimizers.

Default 1.0e-7

Accepted [1.0e-12; +inf]

Groups *Simplex optimizer*

MSK_DPAR_UPPER_OBJ_CUT

If either a primal or dual feasible solution is found proving that the optimal objective value is outside, the interval [*MSK_DPAR_LOWER_OBJ_CUT* , *MSK_DPAR_UPPER_OBJ_CUT*], then **MOSEK** is terminated.

Default 1.0e30

Accepted [-inf; +inf]

Groups *Termination criteria*

See also *MSK_DPAR_UPPER_OBJ_CUT_FINITE_TRH*

MSK_DPAR_UPPER_OBJ_CUT_FINITE_TRH

If the upper objective cut is greater than the value of this parameter, then the upper objective cut *MSK_DPAR_UPPER_OBJ_CUT* is treated as ∞.

Default 0.5e30

Accepted [-inf; +inf]

Groups *Termination criteria*

12.3.2 Integer parameters

iparam

The enumeration type containing all integer parameters.

MSK_IPAR_ANA_SOL_BASIS

Controls whether the basis matrix is analyzed in solution analyzer.

Default *"ON"*

Accepted *"ON"*, *"OFF"*

Groups *Analysis*

MSK_IPAR_ANA_SOL_PRINT_VIOLATED

Controls whether a list of violated constraints is printed.

All constraints violated by more than the value set by the parameter *MSK_DPAR_ANA_SOL_INFEAS_TOL* will be printed.

>   **Default** *"OFF"*
>
>   **Accepted** *"ON"*, *"OFF"*
>
>   **Groups** *Analysis*

MSK_IPAR_AUTO_SORT_A_BEFORE_OPT

>   Controls whether the elements in each column of A are sorted before an optimization is performed. This is not required but makes the optimization more deterministic.
>
>   **Default** *"OFF"*
>
>   **Accepted** *"ON"*, *"OFF"*
>
>   **Groups** *Debugging*

MSK_IPAR_AUTO_UPDATE_SOL_INFO

>   Controls whether the solution information items are automatically updated after an optimization is performed.
>
>   **Default** *"OFF"*
>
>   **Accepted** *"ON"*, *"OFF"*
>
>   **Groups** *Overall system*

MSK_IPAR_BASIS_SOLVE_USE_PLUS_ONE

>   If a slack variable is in the basis, then the corresponding column in the basis is a unit vector with -1 in the right position. However, if this parameter is set to *"MSK_ON"*, -1 is replaced by 1.
>
>   **Default** *"OFF"*
>
>   **Accepted** *"ON"*, *"OFF"*
>
>   **Groups** *Simplex optimizer*

MSK_IPAR_BI_CLEAN_OPTIMIZER

>   Controls which simplex optimizer is used in the clean-up phase.
>
>   **Default** *"FREE"*
>
>   **Accepted** *"FREE"*, *"INTPNT"*, *"CONIC"*, *"PRIMAL_SIMPLEX"*, *"DUAL_SIMPLEX"*, *"FREE_SIMPLEX"*, *"MIXED_INT"*
>
>   **Groups** *Basis identification*, *Overall solver*

MSK_IPAR_BI_IGNORE_MAX_ITER

>   If the parameter *MSK_IPAR_INTPNT_BASIS* has the value *"MSK_BI_NO_ERROR"* and the interior-point optimizer has terminated due to maximum number of iterations, then basis identification is performed if this parameter has the value *"MSK_ON"*.
>
>   **Default** *"OFF"*
>
>   **Accepted** *"ON"*, *"OFF"*
>
>   **Groups** *Interior-point method*, *Basis identification*

MSK_IPAR_BI_IGNORE_NUM_ERROR

>   If the parameter *MSK_IPAR_INTPNT_BASIS* has the value *"MSK_BI_NO_ERROR"* and the interior-point optimizer has terminated due to a numerical problem, then basis identification is performed if this parameter has the value *"MSK_ON"*.
>
>   **Default** *"OFF"*
>
>   **Accepted** *"ON"*, *"OFF"*
>
>   **Groups** *Interior-point method*, *Basis identification*

MSK_IPAR_BI_MAX_ITERATIONS

>   Controls the maximum number of simplex iterations allowed to optimize a basis after the basis identification.

> **Default** 1000000
>
> **Accepted** [0; +inf]
>
> **Groups** *Basis identification*, *Termination criteria*

MSK_IPAR_CACHE_LICENSE

Specifies if the license is kept checked out for the lifetime of the mosek environment (`"MSK_ON"`) or returned to the server immediately after the optimization (`"MSK_OFF"`).

By default the license is checked out for the lifetime of the **MOSEK** environment by the first call to the optimizer.

Check-in and check-out of licenses have an overhead. Frequent communication with the license server should be avoided.

> **Default** `"ON"`
>
> **Accepted** `"ON"`, `"OFF"`
>
> **Groups** *License manager*

MSK_IPAR_CHECK_CONVEXITY

Specify the level of convexity check on quadratic problems.

> **Default** `"FULL"`
>
> **Accepted** `"NONE"`, `"SIMPLE"`, `"FULL"`
>
> **Groups** *Data check*, *Nonlinear convex method*

MSK_IPAR_COMPRESS_STATFILE

Control compression of stat files.

> **Default** `"ON"`
>
> **Accepted** `"ON"`, `"OFF"`

MSK_IPAR_INFEAS_GENERIC_NAMES

Controls whether generic names are used when an infeasible subproblem is created.

> **Default** `"OFF"`
>
> **Accepted** `"ON"`, `"OFF"`
>
> **Groups** *Infeasibility report*

MSK_IPAR_INFEAS_PREFER_PRIMAL

If both certificates of primal and dual infeasibility are supplied then only the primal is used when this option is turned on.

> **Default** `"ON"`
>
> **Accepted** `"ON"`, `"OFF"`
>
> **Groups** *Overall solver*

MSK_IPAR_INFEAS_REPORT_AUTO

Controls whether an infeasibility report is automatically produced after the optimization if the problem is primal or dual infeasible.

> **Default** `"OFF"`
>
> **Accepted** `"ON"`, `"OFF"`
>
> **Groups** *Data input/output*, *Solution input/output*

MSK_IPAR_INFEAS_REPORT_LEVEL

Controls the amount of information presented in an infeasibility report. Higher values imply more information.

> **Default** 1

Accepted [0; +inf]

Groups *Infeasibility report*, *Output information*

MSK_IPAR_INTPNT_BASIS
Controls whether the interior-point optimizer also computes an optimal basis.

Default *"ALWAYS"*

Accepted *"NEVER"*, *"ALWAYS"*, *"NO_ERROR"*, *"IF_FEASIBLE"*, *"RESERVED"*

Groups *Interior-point method*, *Basis identification*

See also *MSK_IPAR_BI_IGNORE_MAX_ITER*, *MSK_IPAR_BI_IGNORE_NUM_ERROR*, *MSK_IPAR_BI_MAX_ITERATIONS*, *MSK_IPAR_BI_CLEAN_OPTIMIZER*

MSK_IPAR_INTPNT_DIFF_STEP
Controls whether different step sizes are allowed in the primal and dual space.

Default *"ON"*

Accepted

- *"ON"*: Different step sizes are allowed.
- *"OFF"*: Different step sizes are not allowed.

Groups *Interior-point method*

MSK_IPAR_INTPNT_HOTSTART
Currently not in use.

Default *"NONE"*

Accepted *"NONE"*, *"PRIMAL"*, *"DUAL"*, *"PRIMAL_DUAL"*

Groups *Interior-point method*

MSK_IPAR_INTPNT_MAX_ITERATIONS
Controls the maximum number of iterations allowed in the interior-point optimizer.

Default 400

Accepted [0; +inf]

Groups *Interior-point method*, *Termination criteria*

MSK_IPAR_INTPNT_MAX_NUM_COR
Controls the maximum number of correctors allowed by the multiple corrector procedure. A negative value means that **MOSEK** is making the choice.

Default -1

Accepted [-1; +inf]

Groups *Interior-point method*

MSK_IPAR_INTPNT_MAX_NUM_REFINEMENT_STEPS
Maximum number of steps to be used by the iterative refinement of the search direction. A negative value implies that the optimizer chooses the maximum number of iterative refinement steps.

Default -1

Accepted [-inf; +inf]

Groups *Interior-point method*

MSK_IPAR_INTPNT_MULTI_THREAD
Controls whether the interior-point optimizers are allowed to employ multiple threads if more threads is available.

Default *"ON"*

Accepted *"ON"*, *"OFF"*

Groups *Overall system*

`MSK_IPAR_INTPNT_OFF_COL_TRH`
Controls how many offending columns are detected in the Jacobian of the constraint matrix.

0	no detection
1	aggressive detection
> 1	higher values mean less aggressive detection

Default 40

Accepted [0; +inf]

Groups *Interior-point method*

`MSK_IPAR_INTPNT_ORDER_METHOD`
Controls the ordering strategy used by the interior-point optimizer when factorizing the Newton equation system.

Default *"FREE"*

Accepted *"FREE"*, *"APPMINLOC"*, *"EXPERIMENTAL"*, *"TRY_GRAPHPAR"*, *"FORCE_GRAPHPAR"*, *"NONE"*

Groups *Interior-point method*

`MSK_IPAR_INTPNT_REGULARIZATION_USE`
Controls whether regularization is allowed.

Default *"ON"*

Accepted *"ON"*, *"OFF"*

Groups *Interior-point method*

`MSK_IPAR_INTPNT_SCALING`
Controls how the problem is scaled before the interior-point optimizer is used.

Default *"FREE"*

Accepted *"FREE"*, *"NONE"*, *"MODERATE"*, *"AGGRESSIVE"*

Groups *Interior-point method*

`MSK_IPAR_INTPNT_SOLVE_FORM`
Controls whether the primal or the dual problem is solved.

Default *"FREE"*

Accepted *"FREE"*, *"PRIMAL"*, *"DUAL"*

Groups *Interior-point method*

`MSK_IPAR_INTPNT_STARTING_POINT`
Starting point used by the interior-point optimizer.

Default *"FREE"*

Accepted *"FREE"*, *"GUESS"*, *"CONSTANT"*, *"SATISFY_BOUNDS"*

Groups *Interior-point method*

`MSK_IPAR_LICENSE_DEBUG`
This option is used to turn on debugging of the license manager.

Default *"OFF"*

Accepted *"ON"*, *"OFF"*

Groups *License manager*

MSK_IPAR_LICENSE_PAUSE_TIME

> If *MSK_IPAR_LICENSE_WAIT* = *"MSK_ON"* and no license is available, then **MOSEK** sleeps a number of milliseconds between each check of whether a license has become free.
>
> > **Default** 100
> >
> > **Accepted** [0; 1000000]
> >
> > **Groups** *License manager*

MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRNS

> Controls whether license features expire warnings are suppressed.
>
> > **Default** *"OFF"*
> >
> > **Accepted** *"ON"*, *"OFF"*
> >
> > **Groups** *License manager*, *Output information*

MSK_IPAR_LICENSE_TRH_EXPIRY_WRN

> If a license feature expires in a numbers days less than the value of this parameter then a warning will be issued.
>
> > **Default** 7
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *License manager*, *Output information*

MSK_IPAR_LICENSE_WAIT

> If all licenses are in use **MOSEK** returns with an error code. However, by turning on this parameter **MOSEK** will wait for an available license.
>
> > **Default** *"OFF"*
> >
> > **Accepted** *"ON"*, *"OFF"*
> >
> > **Groups** *Overall solver*, *Overall system*, *License manager*

MSK_IPAR_LOG

> Controls the amount of log information. The value 0 implies that all log information is suppressed. A higher level implies that more information is logged.
>
> Please note that if a task is employed to solve a sequence of optimization problems the value of this parameter is reduced by the value of *MSK_IPAR_LOG_CUT_SECOND_OPT* for the second and any subsequent optimizations.
>
> > **Default** 10
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Output information*, *Logging*
> >
> > **See also** *MSK_IPAR_LOG_CUT_SECOND_OPT*

MSK_IPAR_LOG_ANA_PRO

> Controls amount of output from the problem analyzer.
>
> > **Default** 1
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Analysis*, *Logging*

MSK_IPAR_LOG_BI

> Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.
>
> > **Default** 1
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Basis identification*, *Output information*, *Logging*

MSK_IPAR_LOG_BI_FREQ
Controls how frequent the optimizer outputs information about the basis identification and how frequent the user-defined callback function is called.

> **Default** 2500
>
> **Accepted** [0; +inf]
>
> **Groups** *Basis identification*, *Output information*, *Logging*

MSK_IPAR_LOG_CHECK_CONVEXITY
Controls logging in convexity check on quadratic problems. Set to a positive value to turn logging on. If a quadratic coefficient matrix is found to violate the requirement of PSD (NSD) then a list of negative (positive) pivot elements is printed. The absolute value of the pivot elements is also shown.

> **Default** 0
>
> **Accepted** [0; +inf]
>
> **Groups** *Data check*, *Nonlinear convex method*

MSK_IPAR_LOG_CUT_SECOND_OPT
If a task is employed to solve a sequence of optimization problems, then the value of the log levels is reduced by the value of this parameter. E.g `MSK_IPAR_LOG` and `MSK_IPAR_LOG_SIM` are reduced by the value of this parameter for the second and any subsequent optimizations.

> **Default** 1
>
> **Accepted** [0; +inf]
>
> **Groups** *Output information*, *Logging*
>
> **See also** `MSK_IPAR_LOG`, `MSK_IPAR_LOG_INTPNT`, `MSK_IPAR_LOG_MIO`, `MSK_IPAR_LOG_SIM`

MSK_IPAR_LOG_EXPAND
Controls the amount of logging when a data item such as the maximum number constrains is expanded.

> **Default** 0
>
> **Accepted** [0; +inf]
>
> **Groups** *Output information*, *Logging*

MSK_IPAR_LOG_FEAS_REPAIR
Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.

> **Default** 1
>
> **Accepted** [0; +inf]
>
> **Groups** *Output information*, *Logging*

MSK_IPAR_LOG_FILE
If turned on, then some log info is printed when a file is written or read.

> **Default** 1
>
> **Accepted** [0; +inf]
>
> **Groups** *Data input/output*, *Output information*, *Logging*

MSK_IPAR_LOG_INFEAS_ANA
Controls amount of output printed by the infeasibility analyzer procedures. A higher level implies that more information is logged.

> **Default** 1
>
> **Accepted** [0; +inf]

> **Groups** *Infeasibility report*, *Output information*, *Logging*

MSK_IPAR_LOG_INTPNT

> Controls amount of output printed by the interior-point optimizer. A higher level implies that more information is logged.
>
> > **Default** 1
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Interior-point method*, *Output information*, *Logging*

MSK_IPAR_LOG_MIO

> Controls the log level for the mixed-integer optimizer. A higher level implies that more information is logged.
>
> > **Default** 4
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Mixed-integer optimization*, *Output information*, *Logging*

MSK_IPAR_LOG_MIO_FREQ

> Controls how frequent the mixed-integer optimizer prints the log line. It will print line every time `MSK_IPAR_LOG_MIO_FREQ` relaxations have been solved.
>
> > **Default** 10
> >
> > **Accepted** [-inf; +inf]
> >
> > **Groups** *Mixed-integer optimization*, *Output information*, *Logging*

MSK_IPAR_LOG_ORDER

> If turned on, then factor lines are added to the log.
>
> > **Default** 1
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Output information*, *Logging*

MSK_IPAR_LOG_PRESOLVE

> Controls amount of output printed by the presolve procedure. A higher level implies that more information is logged.
>
> > **Default** 1
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Logging*

MSK_IPAR_LOG_RESPONSE

> Controls amount of output printed when response codes are reported. A higher level implies that more information is logged.
>
> > **Default** 0
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Output information*, *Logging*

MSK_IPAR_LOG_SENSITIVITY

> Controls the amount of logging during the sensitivity analysis.
>
> > 0.Means no logging information is produced.
> >
> > 1.Timing information is printed.
> >
> > 2.Sensitivity results are printed.
> >
> > **Default** 1
> >
> > **Accepted** [0; +inf]

> **Groups** *Output information*, *Logging*

MSK_IPAR_LOG_SENSITIVITY_OPT
> Controls the amount of logging from the optimizers employed during the sensitivity analysis. 0 means no logging information is produced.
>
> > **Default** 0
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Output information*, *Logging*

MSK_IPAR_LOG_SIM
> Controls amount of output printed by the simplex optimizer. A higher level implies that more information is logged.
>
> > **Default** 4
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Simplex optimizer*, *Output information*, *Logging*

MSK_IPAR_LOG_SIM_FREQ
> Controls how frequent the simplex optimizer outputs information about the optimization and how frequent the user-defined callback function is called.
>
> > **Default** 1000
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Simplex optimizer*, *Output information*, *Logging*

MSK_IPAR_LOG_SIM_MINOR
> Currently not in use.
>
> > **Default** 1
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Simplex optimizer*, *Output information*

MSK_IPAR_LOG_STORAGE
> When turned on, **MOSEK** prints messages regarding the storage usage and allocation.
>
> > **Default** 0
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Output information*, *Overall system*, *Logging*

MSK_IPAR_MAX_NUM_WARNINGS
> Each warning is shown a limit number times controlled by this parameter. A negative value is identical to infinite number of times.
>
> > **Default** 10
> >
> > **Accepted** [-inf; +inf]
> >
> > **Groups** *Output information*

MSK_IPAR_MIO_BRANCH_DIR
> Controls whether the mixed-integer optimizer is branching up or down by default.
>
> > **Default** *"FREE"*
> >
> > **Accepted** *"FREE"*, *"UP"*, *"DOWN"*, *"NEAR"*, *"FAR"*, *"ROOT_LP"*, *"GUIDED"*, *"PSEUDOCOST"*
> >
> > **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_CONSTRUCT_SOL

If set to *"MSK_ON"* and all integer variables have been given a value for which a feasible mixed integer solution exists, then **MOSEK** generates an initial solution to the mixed integer problem by fixing all integer values and solving the remaining problem.

> **Default** *"OFF"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_CUT_CLIQUE

Controls whether clique cuts should be generated.

> **Default** *"ON"*
>
> **Accepted**
>
> - *"ON"*: Turns generation of this cut class on.
> - *"OFF"*: Turns generation of this cut class off.
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_CUT_CMIR

Controls whether mixed integer rounding cuts should be generated.

> **Default** *"ON"*
>
> **Accepted**
>
> - *"ON"*: Turns generation of this cut class on.
> - *"OFF"*: Turns generation of this cut class off.
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_CUT_GMI

Controls whether GMI cuts should be generated.

> **Default** *"ON"*
>
> **Accepted**
>
> - *"ON"*: Turns generation of this cut class on.
> - *"OFF"*: Turns generation of this cut class off.
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_CUT_IMPLIED_BOUND

Controls whether implied bound cuts should be generated.

> **Default** *"OFF"*
>
> **Accepted**
>
> - *"ON"*: Turns generation of this cut class on.
> - *"OFF"*: Turns generation of this cut class off.
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_CUT_KNAPSACK_COVER

Controls whether knapsack cover cuts should be generated.

> **Default** *"OFF"*
>
> **Accepted**
>
> - *"ON"*: Turns generation of this cut class on.
> - *"OFF"*: Turns generation of this cut class off.
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_CUT_SELECTION_LEVEL

Controls how aggressively generated cuts are selected to be included in the relaxation.

-1. The optimizer chooses the level of cut selection

0.Generated cuts less likely to be added to the relaxation

1.Cuts are more aggressively selected to be included in the relaxation

Default -1

Accepted [-1; +1]

Groups *Mixed-integer optimization*

MSK_IPAR_MIO_HEURISTIC_LEVEL

Controls the heuristic employed by the mixed-integer optimizer to locate an initial good integer feasible solution. A value of zero means the heuristic is not used at all. A larger value than 0 means that a gradually more sophisticated heuristic is used which is computationally more expensive. A negative value implies that the optimizer chooses the heuristic. Normally a value around 3 to 5 should be optimal.

Default -1

Accepted [-inf; +inf]

Groups *Mixed-integer optimization*

MSK_IPAR_MIO_MAX_NUM_BRANCHES

Maximum number of branches allowed during the branch and bound search. A negative value means infinite.

Default -1

Accepted [-inf; +inf]

Groups *Mixed-integer optimization*, *Termination criteria*

See also *MSK_DPAR_MIO_DISABLE_TERM_TIME*

MSK_IPAR_MIO_MAX_NUM_RELAXS

Maximum number of relaxations allowed during the branch and bound search. A negative value means infinite.

Default -1

Accepted [-inf; +inf]

Groups *Mixed-integer optimization*

See also *MSK_DPAR_MIO_DISABLE_TERM_TIME*

MSK_IPAR_MIO_MAX_NUM_SOLUTIONS

The mixed-integer optimizer can be terminated after a certain number of different feasible solutions has been located. If this parameter has the value $n > 0$, then the mixed-integer optimizer will be terminated when n feasible solutions have been located.

Default -1

Accepted [-inf; +inf]

Groups *Mixed-integer optimization*, *Termination criteria*

See also *MSK_DPAR_MIO_DISABLE_TERM_TIME*

MSK_IPAR_MIO_MODE

Controls whether the optimizer includes the integer restrictions when solving a (mixed) integer optimization problem.

Default *"SATISFIED"*

> **Accepted** *"IGNORED"*, *"SATISFIED"*
>
> **Groups** *Overall solver*

MSK_IPAR_MIO_MT_USER_CB
If true user callbacks are called from each thread used by mixed-integer optimizer. Otherwise it is only called from a single thread.

> **Default** *"OFF"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Overall system*

MSK_IPAR_MIO_NODE_OPTIMIZER
Controls which optimizer is employed at the non-root nodes in the mixed-integer optimizer.

> **Default** *"FREE"*
>
> **Accepted** *"FREE"*, *"INTPNT"*, *"CONIC"*, *"PRIMAL_SIMPLEX"*, *"DUAL_SIMPLEX"*, *"FREE_SIMPLEX"*, *"MIXED_INT"*
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_NODE_SELECTION
Controls the node selection strategy employed by the mixed-integer optimizer.

> **Default** *"FREE"*
>
> **Accepted** *"FREE"*, *"FIRST"*, *"BEST"*, *"WORST"*, *"HYBRID"*, *"PSEUDO"*
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_PERSPECTIVE_REFORMULATE
Enables or disables perspective reformulation in presolve.

> **Default** *"ON"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_PROBING_LEVEL
Controls the amount of probing employed by the mixed-integer optimizer in presolve.

-1. The optimizer chooses the level of probing employed

0. Probing is disabled

1. A low amount of probing is employed

2. A medium amount of probing is employed

3. A high amount of probing is employed

> **Default** -1
>
> **Accepted** [-1; 3]
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_RINS_MAX_NODES
Controls the maximum number of nodes allowed in each call to the RINS heuristic. The default value of -1 means that the value is determined automatically. A value of zero turns off the heuristic.

> **Default** -1
>
> **Accepted** [-1; +inf]
>
> **Groups** *Mixed-integer optimization*

MSK_IPAR_MIO_ROOT_OPTIMIZER
Controls which optimizer is employed at the root node in the mixed-integer optimizer.

Default *"FREE"*

Accepted *"FREE"*, *"INTPNT"*, *"CONIC"*, *"PRIMAL_SIMPLEX"*, *"DUAL_SIMPLEX"*, *"FREE_SIMPLEX"*, *"MIXED_INT"*

Groups *Mixed-integer optimization*

MSK_IPAR_MIO_ROOT_REPEAT_PRESOLVE_LEVEL
Controls whether presolve can be repeated at root node.

- •-1 The optimizer chooses whether presolve is repeated
- •0 Never repeat presolve
- •1 Always repeat presolve

Default -1

Accepted [-1; 1]

Groups *Mixed-integer optimization*

MSK_IPAR_MIO_VB_DETECTION_LEVEL
Controls how much effort is put into detecting variable bounds.

-1. The optimizer chooses

0.No variable bounds are detected

1.Only detect variable bounds that are directly represented in the problem

2.Detect variable bounds in probing

Default -1

Accepted [-1; +2]

Groups *Mixed-integer optimization*

MSK_IPAR_MT_SPINCOUNT
Set the number of iterations to spin before sleeping.

Default 0

Accepted [0; 1000000000]

Groups *Overall system*

MSK_IPAR_NUM_THREADS
Controls the number of threads employed by the optimizer. If set to 0 the number of threads used will be equal to the number of cores detected on the machine.

Default 0

Accepted [0; +inf]

Groups *Overall system*

MSK_IPAR_OPF_MAX_TERMS_PER_LINE
The maximum number of terms (linear and quadratic) per line when an OPF file is written.

Default 5

Accepted [0; +inf]

Groups *Data input/output*

MSK_IPAR_OPF_WRITE_HEADER
Write a text header with date and **MOSEK** version in an OPF file.

Default *"ON"*

> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*

MSK_IPAR_OPF_WRITE_HINTS
Write a hint section with problem dimensions in the beginning of an OPF file.

> **Default** *"ON"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*

MSK_IPAR_OPF_WRITE_PARAMETERS
Write a parameter section in an OPF file.

> **Default** *"OFF"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*

MSK_IPAR_OPF_WRITE_PROBLEM
Write objective, constraints, bounds etc. to an OPF file.

> **Default** *"ON"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*

MSK_IPAR_OPF_WRITE_SOL_BAS
If *MSK_IPAR_OPF_WRITE_SOLUTIONS* is *"MSK_ON"* and a basic solution is defined, include the basic solution in OPF files.

> **Default** *"ON"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*

MSK_IPAR_OPF_WRITE_SOL_ITG
If *MSK_IPAR_OPF_WRITE_SOLUTIONS* is *"MSK_ON"* and an integer solution is defined, write the integer solution in OPF files.

> **Default** *"ON"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*

MSK_IPAR_OPF_WRITE_SOL_ITR
If *MSK_IPAR_OPF_WRITE_SOLUTIONS* is *"MSK_ON"* and an interior solution is defined, write the interior solution in OPF files.

> **Default** *"ON"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*

MSK_IPAR_OPF_WRITE_SOLUTIONS
Enable inclusion of solutions in the OPF files.

> **Default** *"OFF"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*

MSK_IPAR_OPTIMIZER
The parameter controls which optimizer is used to optimize the task.

> **Default** *"FREE"*

Accepted *"FREE"*, *"INTPNT"*, *"CONIC"*, *"PRIMAL_SIMPLEX"*, *"DUAL_SIMPLEX"*, *"FREE_SIMPLEX"*, *"MIXED_INT"*

Groups *Overall solver*

MSK_IPAR_PARAM_READ_CASE_NAME
If turned on, then names in the parameter file are case sensitive.

Default *"ON"*

Accepted *"ON"*, *"OFF"*

Groups *Data input/output*

MSK_IPAR_PARAM_READ_IGN_ERROR
If turned on, then errors in parameter settings is ignored.

Default *"OFF"*

Accepted *"ON"*, *"OFF"*

Groups *Data input/output*

MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL
Controls the maximum amount of fill-in that can be created by one pivot in the elimination phase of the presolve. A negative value means the parameter value is selected automatically.

Default -1

Accepted [-inf; +inf]

Groups *Presolve*

MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_NUM_TRIES
Control the maximum number of times the eliminator is tried. A negative value implies **MOSEK** decides.

Default -1

Accepted [-inf; +inf]

Groups *Presolve*

MSK_IPAR_PRESOLVE_LEVEL
Currently not used.

Default -1

Accepted [-inf; +inf]

Groups *Overall solver*, *Presolve*

MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH
The linear dependency check is potentially computationally expensive.

Default 100

Accepted [-inf; +inf]

Groups *Presolve*

MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH
The linear dependency check is potentially computationally expensive.

Default 100

Accepted [-inf; +inf]

Groups *Presolve*

MSK_IPAR_PRESOLVE_LINDEP_USE
Controls whether the linear constraints are checked for linear dependencies.

Default *"ON"*

> **Accepted**
>
> - *"ON"*: Turns the linear dependency check on.
>
> - *"OFF"*: Turns the linear dependency check off.
>
> **Groups** *Presolve*

MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS
Controls the maximum number of reductions performed by the presolve. The value of the parameter is normally only changed in connection with debugging. A negative value implies that an infinite number of reductions are allowed.

> **Default** -1
>
> **Accepted** [-inf; +inf]
>
> **Groups** *Overall solver*, *Presolve*

MSK_IPAR_PRESOLVE_USE
Controls whether the presolve is applied to a problem before it is optimized.

> **Default** *"FREE"*
>
> **Accepted** *"OFF"*, *"ON"*, *"FREE"*
>
> **Groups** *Overall solver*, *Presolve*

MSK_IPAR_PRIMAL_REPAIR_OPTIMIZER
Controls which optimizer that is used to find the optimal repair.

> **Default** *"FREE"*
>
> **Accepted** *"FREE"*, *"INTPNT"*, *"CONIC"*, *"PRIMAL_SIMPLEX"*, *"DUAL_SIMPLEX"*, *"FREE_SIMPLEX"*, *"MIXED_INT"*
>
> **Groups** *Overall solver*

MSK_IPAR_READ_DATA_COMPRESSED
If this option is turned on,it is assumed that the data file is compressed.

> **Default** *"FREE"*
>
> **Accepted** *"NONE"*, *"FREE"*, *"GZIP"*
>
> **Groups** *Data input/output*

MSK_IPAR_READ_DATA_FORMAT
Format of the data file to be read.

> **Default** *"EXTENSION"*
>
> **Accepted** *"EXTENSION"*, *"MPS"*, *"LP"*, *"OP"*, *"XML"*, *"FREE_MPS"*, *"TASK"*, *"CB"*, *"JSON_TASK"*
>
> **Groups** *Data input/output*

MSK_IPAR_READ_DEBUG
Turns on additional debugging information when reading files.

> **Default** *"OFF"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*

MSK_IPAR_READ_KEEP_FREE_CON
Controls whether the free constraints are included in the problem.

> **Default** *"OFF"*
>
> **Accepted**
>
> - *"ON"*: The free constraints are kept.

- *"OFF"*: The free constraints are discarded.

   Groups *Data input/output*

MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU
   If this option is turned on, **MOSEK** will drop variables that are defined for the first time in the bounds section.

   Default *"OFF"*

   Accepted *"ON"*, *"OFF"*

   Groups *Data input/output*

MSK_IPAR_READ_LP_QUOTED_NAMES
   If a name is in quotes when reading an LP file, the quotes will be removed.

   Default *"ON"*

   Accepted *"ON"*, *"OFF"*

   Groups *Data input/output*

MSK_IPAR_READ_MPS_FORMAT
   Controls how strictly the MPS file reader interprets the MPS format.

   Default *"FREE"*

   Accepted *"STRICT"*, *"RELAXED"*, *"FREE"*, *"CPLEX"*

   Groups *Data input/output*

MSK_IPAR_READ_MPS_WIDTH
   Controls the maximal number of characters allowed in one line of the MPS file.

   Default 1024

   Accepted [80; +inf]

   Groups *Data input/output*

MSK_IPAR_READ_TASK_IGNORE_PARAM
   Controls whether **MOSEK** should ignore the parameter setting defined in the task file and use the default parameter setting instead.

   Default *"OFF"*

   Accepted *"ON"*, *"OFF"*

   Groups *Data input/output*

MSK_IPAR_REMOVE_UNUSED_SOLUTIONS
   Removes unsued solutions before the optimization is performed.

   Default *"OFF"*

   Accepted *"ON"*, *"OFF"*

   Groups *Overall system*

MSK_IPAR_SENSITIVITY_ALL
   Not applicable.

   Default *"OFF"*

   Accepted *"ON"*, *"OFF"*

   Groups *Overall solver*

MSK_IPAR_SENSITIVITY_OPTIMIZER
   Controls which optimizer is used for optimal partition sensitivity analysis.

   Default *"FREE_SIMPLEX"*

> > **Accepted** *"FREE"*, *"INTPNT"*, *"CONIC"*, *"PRIMAL_SIMPLEX"*, *"DUAL_SIMPLEX"*, *"FREE_SIMPLEX"*, *"MIXED_INT"*
>
> > **Groups** *Overall solver*, *Simplex optimizer*

MSK_IPAR_SENSITIVITY_TYPE
Controls which type of sensitivity analysis is to be performed.

> > **Default** *"BASIS"*
>
> > **Accepted** *"BASIS"*, *"OPTIMAL_PARTITION"*
>
> > **Groups** *Overall solver*

MSK_IPAR_SIM_BASIS_FACTOR_USE
Controls whether an LU factorization of the basis is used in a hot-start. Forcing a refactorization sometimes improves the stability of the simplex optimizers, but in most cases there is a performance penalty.

> > **Default** *"ON"*
>
> > **Accepted** *"ON"*, *"OFF"*
>
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_DEGEN
Controls how aggressively degeneration is handled.

> > **Default** *"FREE"*
>
> > **Accepted** *"NONE"*, *"FREE"*, *"AGGRESSIVE"*, *"MODERATE"*, *"MINIMUM"*
>
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_DUAL_CRASH
Controls whether crashing is performed in the dual simplex optimizer.

If this parameter is set to x, then a crash will be performed if a basis consists of more than $(100-x)$ mod f_v entries, where f_v is the number of fixed variables.

> > **Default** 90
>
> > **Accepted** [0; +inf]
>
> > **Groups** *Dual simplex*

MSK_IPAR_SIM_DUAL_PHASEONE_METHOD
An experimental feature.

> > **Default** 0
>
> > **Accepted** [0; 10]
>
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION
The dual simplex optimizer can use a so-called restricted selection/pricing strategy to chooses the outgoing variable. Hence, if restricted selection is applied, then the dual simplex optimizer first choose a subset of all the potential outgoing variables. Next, for some time it will choose the outgoing variable only among the subset. From time to time the subset is redefined.

A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

> > **Default** 50
>
> > **Accepted** [0; 100]
>
> > **Groups** *Dual simplex*

MSK_IPAR_SIM_DUAL_SELECTION
> Controls the choice of the incoming variable, known as the selection strategy, in the dual simplex optimizer.
>
> > **Default** *"FREE"*
> >
> > **Accepted** *"FREE"*, *"FULL"*, *"ASE"*, *"DEVEX"*, *"SE"*, *"PARTIAL"*
> >
> > **Groups** *Dual simplex*

MSK_IPAR_SIM_EXPLOIT_DUPVEC
> Controls if the simplex optimizers are allowed to exploit duplicated columns.
>
> > **Default** *"OFF"*
> >
> > **Accepted** *"ON"*, *"OFF"*, *"FREE"*
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_HOTSTART
> Controls the type of hot-start that the simplex optimizer perform.
>
> > **Default** *"FREE"*
> >
> > **Accepted** *"NONE"*, *"FREE"*, *"STATUS_KEYS"*
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_HOTSTART_LU
> Determines if the simplex optimizer should exploit the initial factorization.
>
> > **Default** *"ON"*
> >
> > **Accepted**
> >
> > > - *"ON"*: Factorization is reused if possible.
> > > - *"OFF"*: Factorization is recomputed.
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_MAX_ITERATIONS
> Maximum number of iterations that can be used by a simplex optimizer.
>
> > **Default** 10000000
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Simplex optimizer*, *Termination criteria*

MSK_IPAR_SIM_MAX_NUM_SETBACKS
> Controls how many set-backs are allowed within a simplex optimizer. A set-back is an event where the optimizer moves in the wrong direction. This is impossible in theory but may happen due to numerical problems.
>
> > **Default** 250
> >
> > **Accepted** [0; +inf]
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_NON_SINGULAR
> Controls if the simplex optimizer ensures a non-singular basis, if possible.
>
> > **Default** *"ON"*
> >
> > **Accepted** *"ON"*, *"OFF"*
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_PRIMAL_CRASH

Controls whether crashing is performed in the primal simplex optimizer.

In general, if a basis consists of more than (100-this parameter value)% fixed variables, then a crash will be performed.

> **Default** 90
>
> **Accepted** [0; +inf]
>
> **Groups** *Primal simplex*

MSK_IPAR_SIM_PRIMAL_PHASEONE_METHOD

An experimental feature.

> **Default** 0
>
> **Accepted** [0; 10]
>
> **Groups** *Simplex optimizer*

MSK_IPAR_SIM_PRIMAL_RESTRICT_SELECTION

The primal simplex optimizer can use a so-called restricted selection/pricing strategy to chooses the outgoing variable. Hence, if restricted selection is applied, then the primal simplex optimizer first choose a subset of all the potential incoming variables. Next, for some time it will choose the incoming variable only among the subset. From time to time the subset is redefined.

A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

> **Default** 50
>
> **Accepted** [0; 100]
>
> **Groups** *Primal simplex*

MSK_IPAR_SIM_PRIMAL_SELECTION

Controls the choice of the incoming variable, known as the selection strategy, in the primal simplex optimizer.

> **Default** *"FREE"*
>
> **Accepted** *"FREE"*, *"FULL"*, *"ASE"*, *"DEVEX"*, *"SE"*, *"PARTIAL"*
>
> **Groups** *Primal simplex*

MSK_IPAR_SIM_REFACTOR_FREQ

Controls how frequent the basis is refactorized. The value 0 means that the optimizer determines the best point of refactorization.

It is strongly recommended NOT to change this parameter.

> **Default** 0
>
> **Accepted** [0; +inf]
>
> **Groups** *Simplex optimizer*

MSK_IPAR_SIM_REFORMULATION

Controls if the simplex optimizers are allowed to reformulate the problem.

> **Default** *"OFF"*
>
> **Accepted** *"ON"*, *"OFF"*, *"FREE"*, *"AGGRESSIVE"*
>
> **Groups** *Simplex optimizer*

MSK_IPAR_SIM_SAVE_LU

Controls if the LU factorization stored should be replaced with the LU factorization corresponding to the initial basis.

> **Default** *"OFF"*

> > **Accepted** *"ON"*, *"OFF"*
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_SCALING
> Controls how much effort is used in scaling the problem before a simplex optimizer is used.
>
> > **Default** *"FREE"*
> >
> > **Accepted** *"FREE"*, *"NONE"*, *"MODERATE"*, *"AGGRESSIVE"*
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_SCALING_METHOD
> Controls how the problem is scaled before a simplex optimizer is used.
>
> > **Default** *"POW2"*
> >
> > **Accepted** *"POW2"*, *"FREE"*
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_SOLVE_FORM
> Controls whether the primal or the dual problem is solved by the primal-/dual-simplex optimizer.
>
> > **Default** *"FREE"*
> >
> > **Accepted** *"FREE"*, *"PRIMAL"*, *"DUAL"*
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_STABILITY_PRIORITY
> Controls how high priority the numerical stability should be given.
>
> > **Default** 50
> >
> > **Accepted** [0; 100]
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SIM_SWITCH_OPTIMIZER
> The simplex optimizer sometimes chooses to solve the dual problem instead of the primal problem. This implies that if you have chosen to use the dual simplex optimizer and the problem is dualized, then it actually makes sense to use the primal simplex optimizer instead. If this parameter is on and the problem is dualized and furthermore the simplex optimizer is chosen to be the primal (dual) one, then it is switched to the dual (primal).
>
> > **Default** *"OFF"*
> >
> > **Accepted** *"ON"*, *"OFF"*
> >
> > **Groups** *Simplex optimizer*

MSK_IPAR_SOL_FILTER_KEEP_BASIC
> If turned on, then basic and super basic constraints and variables are written to the solution file independent of the filter setting.
>
> > **Default** *"OFF"*
> >
> > **Accepted** *"ON"*, *"OFF"*
> >
> > **Groups** *Solution input/output*

MSK_IPAR_SOL_FILTER_KEEP_RANGED
> If turned on, then ranged constraints and variables are written to the solution file independent of the filter setting.
>
> > **Default** *"OFF"*
> >
> > **Accepted** *"ON"*, *"OFF"*
> >
> > **Groups** *Solution input/output*

MSK_IPAR_SOL_READ_NAME_WIDTH

When a solution is read by **MOSEK** and some constraint, variable or cone names contain blanks, then a maximum name width much be specified. A negative value implies that no name contain blanks.

> **Default** -1
>
> **Accepted** [-inf; +inf]
>
> **Groups** *Data input/output*, *Solution input/output*

MSK_IPAR_SOL_READ_WIDTH

Controls the maximal acceptable width of line in the solutions when read by **MOSEK**.

> **Default** 1024
>
> **Accepted** [80; +inf]
>
> **Groups** *Data input/output*, *Solution input/output*

MSK_IPAR_SOLUTION_CALLBACK

Indicates whether solution callbacks will be performed during the optimization.

> **Default** *"OFF"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Progress callback*, *Overall solver*

MSK_IPAR_TIMING_LEVEL

Controls the amount of timing performed inside **MOSEK**.

> **Default** 1
>
> **Accepted** [0; +inf]
>
> **Groups** *Overall system*

MSK_IPAR_WRITE_BAS_CONSTRAINTS

Controls whether the constraint section is written to the basic solution file.

> **Default** *"ON"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_BAS_HEAD

Controls whether the header section is written to the basic solution file.

> **Default** *"ON"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_BAS_VARIABLES

Controls whether the variables section is written to the basic solution file.

> **Default** *"ON"*
>
> **Accepted** *"ON"*, *"OFF"*
>
> **Groups** *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_DATA_COMPRESSED

Controls whether the data file is compressed while it is written. 0 means no compression while higher values mean more compression.

> **Default** 0
>
> **Accepted** [0; +inf]
>
> **Groups** *Data input/output*

`MSK_IPAR_WRITE_DATA_FORMAT`
> Controls the file format when writing task data to a file.
>
>> **Default** *"EXTENSION"*
>>
>> **Accepted** *"EXTENSION"*, *"MPS"*, *"LP"*, *"OP"*, *"XML"*, *"FREE_MPS"*, *"TASK"*, *"CB"*, *"JSON_TASK"*
>>
>> **Groups** *Data input/output*

`MSK_IPAR_WRITE_DATA_PARAM`
> If this option is turned on the parameter settings are written to the data file as parameters.
>
>> **Default** *"OFF"*
>>
>> **Accepted** *"ON"*, *"OFF"*
>>
>> **Groups** *Data input/output*

`MSK_IPAR_WRITE_FREE_CON`
> Controls whether the free constraints are written to the data file.
>
>> **Default** *"ON"*
>>
>> **Accepted**
>>
>>> - *"ON"*: The free constraints are written.
>>> - *"OFF"*: The free constraints are discarded.
>>
>> **Groups** *Data input/output*

`MSK_IPAR_WRITE_GENERIC_NAMES`
> Controls whether the generic names or user-defined names are used in the data file.
>
>> **Default** *"OFF"*
>>
>> **Accepted**
>>
>>> - *"ON"*: Generic names are used.
>>> - *"OFF"*: Generic names are not used.
>>
>> **Groups** *Data input/output*

`MSK_IPAR_WRITE_GENERIC_NAMES_IO`
> Index origin used in generic names.
>
>> **Default** 1
>>
>> **Accepted** [0; +inf]
>>
>> **Groups** *Data input/output*

`MSK_IPAR_WRITE_IGNORE_INCOMPATIBLE_ITEMS`
> Controls if the writer ignores incompatible problem items when writing files.
>
>> **Default** *"OFF"*
>>
>> **Accepted**
>>
>>> - *"ON"*: Ignore items that cannot be written to the current output file format.
>>> - *"OFF"*: Produce an error if the problem contains items that cannot the written to the current output file format.
>>
>> **Groups** *Data input/output*

`MSK_IPAR_WRITE_INT_CONSTRAINTS`
> Controls whether the constraint section is written to the integer solution file.
>
>> **Default** *"ON"*
>>
>> **Accepted** *"ON"*, *"OFF"*

> **Groups** *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_INT_HEAD
> Controls whether the header section is written to the integer solution file.

>> **Default** *"ON"*

>> **Accepted** *"ON"*, *"OFF"*

>> **Groups** *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_INT_VARIABLES
> Controls whether the variables section is written to the integer solution file.

>> **Default** *"ON"*

>> **Accepted** *"ON"*, *"OFF"*

>> **Groups** *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_LP_FULL_OBJ
> Write all variables, including the ones with 0-coefficients, in the objective.

>> **Default** *"ON"*

>> **Accepted** *"ON"*, *"OFF"*

>> **Groups** *Data input/output*

MSK_IPAR_WRITE_LP_LINE_WIDTH
> Maximum width of line in an LP file written by **MOSEK**.

>> **Default** 80

>> **Accepted** [40; +inf]

>> **Groups** *Data input/output*

MSK_IPAR_WRITE_LP_QUOTED_NAMES
> If this option is turned on, then **MOSEK** will quote invalid LP names when writing an LP file.

>> **Default** *"ON"*

>> **Accepted** *"ON"*, *"OFF"*

>> **Groups** *Data input/output*

MSK_IPAR_WRITE_LP_STRICT_FORMAT
> Controls whether LP output files satisfy the LP format strictly.

>> **Default** *"OFF"*

>> **Accepted** *"ON"*, *"OFF"*

>> **Groups** *Data input/output*

MSK_IPAR_WRITE_LP_TERMS_PER_LINE
> Maximum number of terms on a single line in an LP file written by **MOSEK**. 0 means unlimited.

>> **Default** 10

>> **Accepted** [0; +inf]

>> **Groups** *Data input/output*

MSK_IPAR_WRITE_MPS_FORMAT
> Controls in which format the MPS is written.

>> **Default** *"FREE"*

>> **Accepted** *"STRICT"*, *"RELAXED"*, *"FREE"*, *"CPLEX"*

>> **Groups** *Data input/output*

MSK_IPAR_WRITE_MPS_INT
: Controls if marker records are written to the MPS file to indicate whether variables are integer restricted.

   Default *"ON"*

   Accepted

   - *"ON"*: Marker records are written.
   - *"OFF"*: Marker records are not written.

   Groups *Data input/output*

MSK_IPAR_WRITE_PRECISION
: Controls the precision with which `double` numbers are printed in the MPS data file. In general it is not worthwhile to use a value higher than 15.

   Default 15

   Accepted [0; +inf]

   Groups *Data input/output*

MSK_IPAR_WRITE_SOL_BARVARIABLES
: Controls whether the symmetric matrix variables section is written to the solution file.

   Default *"ON"*

   Accepted *"ON"*, *"OFF"*

   Groups *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_SOL_CONSTRAINTS
: Controls whether the constraint section is written to the solution file.

   Default *"ON"*

   Accepted *"ON"*, *"OFF"*

   Groups *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_SOL_HEAD
: Controls whether the header section is written to the solution file.

   Default *"ON"*

   Accepted *"ON"*, *"OFF"*

   Groups *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_SOL_IGNORE_INVALID_NAMES
: Even if the names are invalid MPS names, then they are employed when writing the solution file.

   Default *"OFF"*

   Accepted *"ON"*, *"OFF"*

   Groups *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_SOL_VARIABLES
: Controls whether the variables section is written to the solution file.

   Default *"ON"*

   Accepted *"ON"*, *"OFF"*

   Groups *Data input/output*, *Solution input/output*

MSK_IPAR_WRITE_TASK_INC_SOL
: Controls whether the solutions are stored in the task file too.

   Default *"ON"*

> > **Accepted** *"ON"*, *"OFF"*
>
> > **Groups** *Data input/output*

MSK_IPAR_WRITE_XML_MODE
> Controls if linear coefficients should be written by row or column when writing in the XML file format.

> > **Default** *"ROW"*

> > **Accepted** *"ROW"*, *"COL"*

> > **Groups** *Data input/output*

12.3.3 String parameters

sparam
> The enumeration type containing all string parameters.

MSK_SPAR_BAS_SOL_FILE_NAME
> Name of the `bas` solution file.

> > **Accepted** Any valid file name.

> > **Groups** *Data input/output*, *Solution input/output*

MSK_SPAR_DATA_FILE_NAME
> Data are read and written to this file.

> > **Accepted** Any valid file name.

> > **Groups** *Data input/output*

MSK_SPAR_DEBUG_FILE_NAME
> **MOSEK** debug file.

> > **Accepted** Any valid file name.

> > **Groups** *Data input/output*

MSK_SPAR_INT_SOL_FILE_NAME
> Name of the `int` solution file.

> > **Accepted** Any valid file name.

> > **Groups** *Data input/output*, *Solution input/output*

MSK_SPAR_ITR_SOL_FILE_NAME
> Name of the `itr` solution file.

> > **Accepted** Any valid file name.

> > **Groups** *Data input/output*, *Solution input/output*

MSK_SPAR_MIO_DEBUG_STRING
> For internal debugging purposes.

> > **Accepted** Any valid string.

> > **Groups** *Data input/output*

MSK_SPAR_PARAM_COMMENT_SIGN
> Only the first character in this string is used. It is considered as a start of comment sign in the **MOSEK** parameter file. Spaces are ignored in the string.

> > **Default**

> > > %%

> > **Accepted** Any valid string.

> **Groups** *Data input/output*

MSK_SPAR_PARAM_READ_FILE_NAME
> Modifications to the parameter database is read from this file.
>
> > **Accepted** Any valid file name.
> >
> > **Groups** *Data input/output*

MSK_SPAR_PARAM_WRITE_FILE_NAME
> The parameter database is written to this file.
>
> > **Accepted** Any valid file name.
> >
> > **Groups** *Data input/output*

MSK_SPAR_READ_MPS_BOU_NAME
> Name of the BOUNDS vector used. An empty name means that the first BOUNDS vector is used.
>
> > **Accepted** Any valid MPS name.
> >
> > **Groups** *Data input/output*

MSK_SPAR_READ_MPS_OBJ_NAME
> Name of the free constraint used as objective function. An empty name means that the first constraint is used as objective function.
>
> > **Accepted** Any valid MPS name.
> >
> > **Groups** *Data input/output*

MSK_SPAR_READ_MPS_RAN_NAME
> Name of the RANGE vector used. An empty name means that the first RANGE vector is used.
>
> > **Accepted** Any valid MPS name.
> >
> > **Groups** *Data input/output*

MSK_SPAR_READ_MPS_RHS_NAME
> Name of the RHS used. An empty name means that the first RHS vector is used.
>
> > **Accepted** Any valid MPS name.
> >
> > **Groups** *Data input/output*

MSK_SPAR_REMOTE_ACCESS_TOKEN
> An access token used to submit tasks to a remote **MOSEK** server. An access token is a random 32-byte string encoded in base64, i.e. it is a 44 character ASCII string.
>
> > **Accepted** Any valid string.
> >
> > **Groups** *Overall system*

MSK_SPAR_SENSITIVITY_FILE_NAME
> If defined, **MOSEK** reads this file as a sensitivity analysis data file specifying the type of analysis to be done.
>
> > **Accepted** Any valid string.
> >
> > **Groups** *Data input/output*

MSK_SPAR_SENSITIVITY_RES_FILE_NAME
> > **Accepted** Any valid string.
> >
> > **Groups** *Data input/output*

MSK_SPAR_SOL_FILTER_XC_LOW
> A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having `xc[i]>0.5` should be listed, whereas +0.5 means that all constraints having `xc[i]>=blc[i]+0.5` should be listed. An empty filter means that no filter is applied.
>
> > **Accepted** Any valid filter.

> **Groups** *Data input/output*, *Solution input/output*

MSK_SPAR_SOL_FILTER_XC_UPR
A filter used to determine which constraints should be listed in the solution file. A value of `0.5` means that all constraints having `xc[i]<0.5` should be listed, whereas `-0.5` means all constraints having `xc[i]<=buc[i]-0.5` should be listed. An empty filter means that no filter is applied.

> **Accepted** Any valid filter.

> **Groups** *Data input/output*, *Solution input/output*

MSK_SPAR_SOL_FILTER_XX_LOW
A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having `xx[j]>=0.5` should be listed, whereas "+0.5" means that all constraints having `xx[j]>=blx[j]+0.5` should be listed. An empty filter means no filter is applied.

> **Accepted** Any valid filter.

> **Groups** *Data input/output*, *Solution input/output*

MSK_SPAR_SOL_FILTER_XX_UPR
A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having `xx[j]<0.5` should be printed, whereas "-0.5" means all constraints having `xx[j]<=bux[j]-0.5` should be listed. An empty filter means no filter is applied.

> **Accepted** Any valid file name.

> **Groups** *Data input/output*, *Solution input/output*

MSK_SPAR_STAT_FILE_NAME
Statistics file name.

> **Accepted** Any valid file name.

> **Groups** *Data input/output*

MSK_SPAR_STAT_KEY
Key used when writing the summary file.

> **Accepted** Any valid string.

> **Groups** *Data input/output*

MSK_SPAR_STAT_NAME
Name used when writing the statistics file.

> **Accepted** Any valid XML string.

> **Groups** *Data input/output*

MSK_SPAR_WRITE_LP_GEN_VAR_NAME
Sometimes when an LP file is written additional variables must be inserted. They will have the prefix denoted by this parameter.

> **Default** xmskgen

> **Accepted** Any valid string.

> **Groups** *Data input/output*

12.4 Response codes

- *Termination*
- *Warnings*
- *Errors*

`rescode`
>   The enumeration type containing all response codes.

12.4.1 Termination

`"MSK_RES_OK"`
>   No error occurred.

`"MSK_RES_TRM_MAX_ITERATIONS"`
>   The optimizer terminated at the maximum number of iterations.

`"MSK_RES_TRM_MAX_TIME"`
>   The optimizer terminated at the maximum amount of time.

`"MSK_RES_TRM_OBJECTIVE_RANGE"`
>   The optimizer terminated with an objective value outside the objective range.

`"MSK_RES_TRM_MIO_NEAR_REL_GAP"`
>   The mixed-integer optimizer terminated as the delayed near optimal relative gap tolerance was satisfied.

`"MSK_RES_TRM_MIO_NEAR_ABS_GAP"`
>   The mixed-integer optimizer terminated as the delayed near optimal absolute gap tolerance was satisfied.

`"MSK_RES_TRM_MIO_NUM_RELAXS"`
>   The mixed-integer optimizer terminated as the maximum number of relaxations was reached.

`"MSK_RES_TRM_MIO_NUM_BRANCHES"`
>   The mixed-integer optimizer terminated as the maximum number of branches was reached.

`"MSK_RES_TRM_NUM_MAX_NUM_INT_SOLUTIONS"`
>   The mixed-integer optimizer terminated as the maximum number of feasible solutions was reached.

`"MSK_RES_TRM_STALL"`
>   The optimizer is terminated due to slow progress.
>
>   Stalling means that numerical problems prevent the optimizer from making reasonable progress and that it make no sense to continue. In many cases this happens if the problem is badly scaled or otherwise ill-conditioned. There is no guarantee that the solution will be (near) feasible or near optimal. However, often stalling happens near the optimum, and the returned solution may be of good quality. Therefore, it is recommended to check the status of then solution. If the solution near optimal the solution is most likely good enough for most practical purposes.
>
>   Please note that if a linear optimization problem is solved using the interior-point optimizer with basis identification turned on, the returned basic solution likely to have high accuracy, even though the optimizer stalled.
>
>   Some common causes of stalling are a) badly scaled models, b) near feasible or near infeasible problems and c) a non-convex problems. Case c) is only relevant for general non-linear problems. It is not possible in general for **MOSEK** to check if a specific problems is convex since such a check would be NP hard in itself. This implies that care should be taken when solving problems involving general user defined functions.

`"MSK_RES_TRM_USER_CALLBACK"`
>   The optimizer terminated due to the return of the user-defined callback function.

`"MSK_RES_TRM_MAX_NUM_SETBACKS"`
>   The optimizer terminated as the maximum number of set-backs was reached. This indicates serious numerical problems and a possibly badly formulated problem.

`"MSK_RES_TRM_NUMERICAL_PROBLEM"`
>   The optimizer terminated due to numerical problems.

`"MSK_RES_TRM_INTERNAL"`
    The optimizer terminated due to some internal reason. Please contact **MOSEK** support.

`"MSK_RES_TRM_INTERNAL_STOP"`
    The optimizer terminated for internal reasons. Please contact **MOSEK** support.

12.4.2 Warnings

`"MSK_RES_WRN_OPEN_PARAM_FILE"`
    The parameter file could not be opened.

`"MSK_RES_WRN_LARGE_BOUND"`
    A numerically large bound value is specified.

`"MSK_RES_WRN_LARGE_LO_BOUND"`
    A numerically large lower bound value is specified.

`"MSK_RES_WRN_LARGE_UP_BOUND"`
    A numerically large upper bound value is specified.

`"MSK_RES_WRN_LARGE_CON_FX"`
    An equality constraint is fixed to a numerically large value. This can cause numerical problems.

`"MSK_RES_WRN_LARGE_CJ"`
    A numerically large value is specified for one c_j.

`"MSK_RES_WRN_LARGE_AIJ"`
    A numerically large value is specified for an $a_{i,j}$ element in A. The parameter
    MSK_DPAR_DATA_TOL_AIJ_LARGE controls when an $a_{i,j}$ is considered large.

`"MSK_RES_WRN_ZERO_AIJ"`
    One or more zero elements are specified in A.

`"MSK_RES_WRN_NAME_MAX_LEN"`
    A name is longer than the buffer that is supposed to hold it.

`"MSK_RES_WRN_SPAR_MAX_LEN"`
    A value for a string parameter is longer than the buffer that is supposed to hold it.

`"MSK_RES_WRN_MPS_SPLIT_RHS_VECTOR"`
    An RHS vector is split into several nonadjacent parts in an MPS file.

`"MSK_RES_WRN_MPS_SPLIT_RAN_VECTOR"`
    A RANGE vector is split into several nonadjacent parts in an MPS file.

`"MSK_RES_WRN_MPS_SPLIT_BOU_VECTOR"`
    A BOUNDS vector is split into several nonadjacent parts in an MPS file.

`"MSK_RES_WRN_LP_OLD_QUAD_FORMAT"`
    Missing '/2' after quadratic expressions in bound or objective.

`"MSK_RES_WRN_LP_DROP_VARIABLE"`
    Ignored a variable because the variable was not previously defined. Usually this implies that a
    variable appears in the bound section but not in the objective or the constraints.

`"MSK_RES_WRN_NZ_IN_UPR_TRI"`
    Non-zero elements specified in the upper triangle of a matrix were ignored.

`"MSK_RES_WRN_DROPPED_NZ_QOBJ"`
    One or more non-zero elements were dropped in the Q matrix in the objective.

`"MSK_RES_WRN_IGNORE_INTEGER"`
    Ignored integer constraints.

`"MSK_RES_WRN_NO_GLOBAL_OPTIMIZER"`
    No global optimizer is available.

"MSK_RES_WRN_MIO_INFEASIBLE_FINAL"
The final mixed-integer problem with all the integer variables fixed at their optimal values is infeasible.

"MSK_RES_WRN_SOL_FILTER"
Invalid solution filter is specified.

"MSK_RES_WRN_UNDEF_SOL_FILE_NAME"
Undefined name occurred in a solution.

"MSK_RES_WRN_SOL_FILE_IGNORED_CON"
One or more lines in the constraint section were ignored when reading a solution file.

"MSK_RES_WRN_SOL_FILE_IGNORED_VAR"
One or more lines in the variable section were ignored when reading a solution file.

"MSK_RES_WRN_TOO_FEW_BASIS_VARS"
An incomplete basis has been specified. Too few basis variables are specified.

"MSK_RES_WRN_TOO_MANY_BASIS_VARS"
A basis with too many variables has been specified.

"MSK_RES_WRN_NO_NONLINEAR_FUNCTION_WRITE"
The problem contains a general nonlinear function in either the objective or the constraints. Such a nonlinear function cannot be written to a disk file. Note that quadratic terms when inputted explicitly can be written to disk.

"MSK_RES_WRN_LICENSE_EXPIRE"
The license expires.

"MSK_RES_WRN_LICENSE_SERVER"
The license server is not responding.

"MSK_RES_WRN_EMPTY_NAME"
A variable or constraint name is empty. The output file may be invalid.

"MSK_RES_WRN_USING_GENERIC_NAMES"
Generic names are used because a name is not valid. For instance when writing an LP file the names must not contain blanks or start with a digit.

"MSK_RES_WRN_LICENSE_FEATURE_EXPIRE"
The license expires.

"MSK_RES_WRN_PARAM_NAME_DOU"
The parameter name is not recognized as a double parameter.

"MSK_RES_WRN_PARAM_NAME_INT"
The parameter name is not recognized as a integer parameter.

"MSK_RES_WRN_PARAM_NAME_STR"
The parameter name is not recognized as a string parameter.

"MSK_RES_WRN_PARAM_STR_VALUE"
The string is not recognized as a symbolic value for the parameter.

"MSK_RES_WRN_PARAM_IGNORED_CMIO"
A parameter was ignored by the conic mixed integer optimizer.

"MSK_RES_WRN_ZEROS_IN_SPARSE_ROW"
One or more (near) zero elements are specified in a sparse row of a matrix. Since, it is redundant to specify zero elements then it may indicate an error.

"MSK_RES_WRN_ZEROS_IN_SPARSE_COL"
One or more (near) zero elements are specified in a sparse column of a matrix. It is redundant to specify zero elements. Hence, it may indicate an error.

"MSK_RES_WRN_INCOMPLETE_LINEAR_DEPENDENCY_CHECK"
The linear dependency check(s) is incomplete. Normally this is not an important warning unless

the optimization problem has been formulated with linear dependencies. Linear dependencies may prevent **MOSEK** from solving the problem.

"MSK_RES_WRN_ELIMINATOR_SPACE"
> The eliminator is skipped at least once due to lack of space.

"MSK_RES_WRN_PRESOLVE_OUTOFSPACE"
> The presolve is incomplete due to lack of space.

"MSK_RES_WRN_WRITE_CHANGED_NAMES"
> Some names were changed because they were invalid for the output file format.

"MSK_RES_WRN_WRITE_DISCARDED_CFIX"
> The fixed objective term could not be converted to a variable and was discarded in the output file.

"MSK_RES_WRN_CONSTRUCT_SOLUTION_INFEAS"
> After fixing the integer variables at the suggested values then the problem is infeasible.

"MSK_RES_WRN_CONSTRUCT_INVALID_SOL_ITG"
> The initial value for one or more of the integer variables is not feasible.

"MSK_RES_WRN_CONSTRUCT_NO_SOL_ITG"
> The construct solution requires an integer solution.

"MSK_RES_WRN_DUPLICATE_CONSTRAINT_NAMES"
> Two constraint names are identical.

"MSK_RES_WRN_DUPLICATE_VARIABLE_NAMES"
> Two variable names are identical.

"MSK_RES_WRN_DUPLICATE_BARVARIABLE_NAMES"
> Two barvariable names are identical.

"MSK_RES_WRN_DUPLICATE_CONE_NAMES"
> Two cone names are identical.

"MSK_RES_WRN_ANA_LARGE_BOUNDS"
> This warning is issued by the problem analyzer, if one or more constraint or variable bounds are very large. One should consider omitting these bounds entirely by setting them to +inf or -inf.

"MSK_RES_WRN_ANA_C_ZERO"
> This warning is issued by the problem analyzer, if the coefficients in the linear part of the objective are all zero.

"MSK_RES_WRN_ANA_EMPTY_COLS"
> This warning is issued by the problem analyzer, if columns, in which all coefficients are zero, are found.

"MSK_RES_WRN_ANA_CLOSE_BOUNDS"
> This warning is issued by problem analyzer, if ranged constraints or variables with very close upper and lower bounds are detected. One should consider treating such constraints as equalities and such variables as constants.

"MSK_RES_WRN_ANA_ALMOST_INT_BOUNDS"
> This warning is issued by the problem analyzer if a constraint is bound nearly integral.

"MSK_RES_WRN_QUAD_CONES_WITH_ROOT_FIXED_AT_ZERO"
> For at least one quadratic cone the root is fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problems, or to fix all the variables in the cone to 0.

"MSK_RES_WRN_RQUAD_CONES_WITH_ROOT_FIXED_AT_ZERO"
> For at least one rotated quadratic cone at least one of the root variables are fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problems, or to fix all the variables in the cone to 0.

"MSK_RES_WRN_NO_DUALIZER"
> No automatic dualizer is available for the specified problem. The primal problem is solved.

`"MSK_RES_WRN_SYM_MAT_LARGE"`
> A numerically large value is specified for an $e_{i,j}$ element in E. The parameter
> `MSK_DPAR_DATA_SYM_MAT_TOL_LARGE` controls when an $e_{i,j}$ is considered large.

12.4.3 Errors

`"MSK_RES_ERR_LICENSE"`
> Invalid license.

`"MSK_RES_ERR_LICENSE_EXPIRED"`
> The license has expired.

`"MSK_RES_ERR_LICENSE_VERSION"`
> The license is valid for another version of **MOSEK**.

`"MSK_RES_ERR_SIZE_LICENSE"`
> The problem is bigger than the license.

`"MSK_RES_ERR_PROB_LICENSE"`
> The software is not licensed to solve the problem.

`"MSK_RES_ERR_FILE_LICENSE"`
> Invalid license file.

`"MSK_RES_ERR_MISSING_LICENSE_FILE"`
> **MOSEK** cannot license file or a token server. See the **MOSEK** installation manual for details.

`"MSK_RES_ERR_SIZE_LICENSE_CON"`
> The problem has too many constraints to be solved with the available license.

`"MSK_RES_ERR_SIZE_LICENSE_VAR"`
> The problem has too many variables to be solved with the available license.

`"MSK_RES_ERR_SIZE_LICENSE_INTVAR"`
> The problem contains too many integer variables to be solved with the available license.

`"MSK_RES_ERR_OPTIMIZER_LICENSE"`
> The optimizer required is not licensed.

`"MSK_RES_ERR_FLEXLM"`
> The FLEXlm license manager reported an error.

`"MSK_RES_ERR_LICENSE_SERVER"`
> The license server is not responding.

`"MSK_RES_ERR_LICENSE_MAX"`
> Maximum number of licenses is reached.

`"MSK_RES_ERR_LICENSE_MOSEKLM_DAEMON"`
> The MOSEKLM license manager daemon is not up and running.

`"MSK_RES_ERR_LICENSE_FEATURE"`
> A requested feature is not available in the license file(s). Most likely due to an incorrect license system setup.

`"MSK_RES_ERR_PLATFORM_NOT_LICENSED"`
> A requested license feature is not available for the required platform.

`"MSK_RES_ERR_LICENSE_CANNOT_ALLOCATE"`
> The license system cannot allocate the memory required.

`"MSK_RES_ERR_LICENSE_CANNOT_CONNECT"`
> **MOSEK** cannot connect to the license server. Most likely the license server is not up and running.

`"MSK_RES_ERR_LICENSE_INVALID_HOSTID"`
> The host ID specified in the license file does not match the host ID of the computer.

"MSK_RES_ERR_LICENSE_SERVER_VERSION"
> The version specified in the checkout request is greater than the highest version number the daemon supports.

"MSK_RES_ERR_LICENSE_NO_SERVER_SUPPORT"
> The license server does not support the requested feature. Possible reasons for this error include:
>
> - •The feature has expired.
>
> - •The feature's start date is later than today's date.
>
> - •The version requested is higher than feature's the highest supported version.
>
> - •A corrupted license file.
>
> Try restarting the license and inspect the license server debug file, usually called `lmgrd.log`.

"MSK_RES_ERR_LICENSE_NO_SERVER_LINE"
> There is no `SERVER` line in the license file. All non-zero license count features need at least one `SERVER` line.

"MSK_RES_ERR_OPEN_DL"
> A dynamic link library could not be opened.

"MSK_RES_ERR_OLDER_DLL"
> The dynamic link library is older than the specified version.

"MSK_RES_ERR_NEWER_DLL"
> The dynamic link library is newer than the specified version.

"MSK_RES_ERR_LINK_FILE_DLL"
> A file cannot be linked to a stream in the DLL version.

"MSK_RES_ERR_THREAD_MUTEX_INIT"
> Could not initialize a mutex.

"MSK_RES_ERR_THREAD_MUTEX_LOCK"
> Could not lock a mutex.

"MSK_RES_ERR_THREAD_MUTEX_UNLOCK"
> Could not unlock a mutex.

"MSK_RES_ERR_THREAD_CREATE"
> Could not create a thread. This error may occur if a large number of environments are created and not deleted again. In any case it is a good practice to minimize the number of environments created.

"MSK_RES_ERR_THREAD_COND_INIT"
> Could not initialize a condition.

"MSK_RES_ERR_UNKNOWN"
> Unknown error.

"MSK_RES_ERR_SPACE"
> Out of space.

"MSK_RES_ERR_FILE_OPEN"
> Error while opening a file.

"MSK_RES_ERR_FILE_READ"
> File read error.

"MSK_RES_ERR_FILE_WRITE"
> File write error.

"MSK_RES_ERR_DATA_FILE_EXT"
> The data file format cannot be determined from the file name.

"MSK_RES_ERR_INVALID_FILE_NAME"
> An invalid file name has been specified.

`"MSK_RES_ERR_INVALID_SOL_FILE_NAME"`
    An invalid file name has been specified.

`"MSK_RES_ERR_END_OF_FILE"`
    End of file reached.

`"MSK_RES_ERR_NULL_ENV"`
    `env` is a NULL pointer.

`"MSK_RES_ERR_NULL_TASK"`
    `task` is a NULL pointer.

`"MSK_RES_ERR_INVALID_STREAM"`
    An invalid stream is referenced.

`"MSK_RES_ERR_NO_INIT_ENV"`
    `env` is not initialized.

`"MSK_RES_ERR_INVALID_TASK"`
    The `task` is invalid.

`"MSK_RES_ERR_NULL_POINTER"`
    An argument to a function is unexpectedly a NULL pointer.

`"MSK_RES_ERR_LIVING_TASKS"`
    All tasks associated with an enviroment must be deleted before the environment is deleted. There
    are still some undeleted tasks.

`"MSK_RES_ERR_BLANK_NAME"`
    An all blank name has been specified.

`"MSK_RES_ERR_DUP_NAME"`
    The same name was used multiple times for the same problem item type.

`"MSK_RES_ERR_INVALID_OBJ_NAME"`
    An invalid objective name is specified.

`"MSK_RES_ERR_INVALID_CON_NAME"`
    An invalid constraint name is used.

`"MSK_RES_ERR_INVALID_VAR_NAME"`
    An invalid variable name is used.

`"MSK_RES_ERR_INVALID_CONE_NAME"`
    An invalid cone name is used.

`"MSK_RES_ERR_INVALID_BARVAR_NAME"`
    An invalid symmetric matrix variable name is used.

`"MSK_RES_ERR_SPACE_LEAKING"`
    MOSEK is leaking memory. This can be due to either an incorrect use of **MOSEK** or a bug.

`"MSK_RES_ERR_SPACE_NO_INFO"`
    No available information about the space usage.

`"MSK_RES_ERR_READ_FORMAT"`
    The specified format cannot be read.

`"MSK_RES_ERR_MPS_FILE"`
    An error occurred while reading an MPS file.

`"MSK_RES_ERR_MPS_INV_FIELD"`
    A field in the MPS file is invalid. Probably it is too wide.

`"MSK_RES_ERR_MPS_INV_MARKER"`
    An invalid marker has been specified in the MPS file.

`"MSK_RES_ERR_MPS_NULL_CON_NAME"`
    An empty constraint name is used in an MPS file.

`"MSK_RES_ERR_MPS_NULL_VAR_NAME"`
> An empty variable name is used in an MPS file.

`"MSK_RES_ERR_MPS_UNDEF_CON_NAME"`
> An undefined constraint name occurred in an MPS file.

`"MSK_RES_ERR_MPS_UNDEF_VAR_NAME"`
> An undefined variable name occurred in an MPS file.

`"MSK_RES_ERR_MPS_INV_CON_KEY"`
> An invalid constraint key occurred in an MPS file.

`"MSK_RES_ERR_MPS_INV_BOUND_KEY"`
> An invalid bound key occurred in an MPS file.

`"MSK_RES_ERR_MPS_INV_SEC_NAME"`
> An invalid section name occurred in an MPS file.

`"MSK_RES_ERR_MPS_NO_OBJECTIVE"`
> No objective is defined in an MPS file.

`"MSK_RES_ERR_MPS_SPLITTED_VAR"`
> All elements in a column of the A matrix must be specified consecutively. Hence, it is illegal to specify non-zero elements in A for variable 1, then for variable 2 and then variable 1 again.

`"MSK_RES_ERR_MPS_MUL_CON_NAME"`
> A constraint name was specified multiple times in the `ROWS` section.

`"MSK_RES_ERR_MPS_MUL_QSEC"`
> Multiple `QSECTION`s are specified for a constraint in the MPS data file.

`"MSK_RES_ERR_MPS_MUL_QOBJ"`
> The Q term in the objective is specified multiple times in the MPS data file.

`"MSK_RES_ERR_MPS_INV_SEC_ORDER"`
> The sections in the MPS data file are not in the correct order.

`"MSK_RES_ERR_MPS_MUL_CSEC"`
> Multiple `CSECTION`s are given the same name.

`"MSK_RES_ERR_MPS_CONE_TYPE"`
> Invalid cone type specified in a `CSECTION`.

`"MSK_RES_ERR_MPS_CONE_OVERLAP"`
> A variable is specified to be a member of several cones.

`"MSK_RES_ERR_MPS_CONE_REPEAT"`
> A variable is repeated within the `CSECTION`.

`"MSK_RES_ERR_MPS_NON_SYMMETRIC_Q"`
> A non symmetric matrice has been speciefied.

`"MSK_RES_ERR_MPS_DUPLICATE_Q_ELEMENT"`
> Duplicate elements is specfied in a Q matrix.

`"MSK_RES_ERR_MPS_INVALID_OBJSENSE"`
> An invalid objective sense is specified.

`"MSK_RES_ERR_MPS_TAB_IN_FIELD2"`
> A tab char occurred in field 2.

`"MSK_RES_ERR_MPS_TAB_IN_FIELD3"`
> A tab char occurred in field 3.

`"MSK_RES_ERR_MPS_TAB_IN_FIELD5"`
> A tab char occurred in field 5.

`"MSK_RES_ERR_MPS_INVALID_OBJ_NAME"`
> An invalid objective name is specified.

12.4. Response codes

`"MSK_RES_ERR_LP_INCOMPATIBLE"`
    The problem cannot be written to an LP formatted file.

`"MSK_RES_ERR_LP_EMPTY"`
    The problem cannot be written to an LP formatted file.

`"MSK_RES_ERR_LP_DUP_SLACK_NAME"`
    The name of the slack variable added to a ranged constraint already exists.

`"MSK_RES_ERR_WRITE_MPS_INVALID_NAME"`
    An invalid name is created while writing an MPS file. Usually this will make the MPS file unreadable.

`"MSK_RES_ERR_LP_INVALID_VAR_NAME"`
    A variable name is invalid when used in an LP formatted file.

`"MSK_RES_ERR_LP_FREE_CONSTRAINT"`
    Free constraints cannot be written in LP file format.

`"MSK_RES_ERR_WRITE_OPF_INVALID_VAR_NAME"`
    Empty variable names cannot be written to OPF files.

`"MSK_RES_ERR_LP_FILE_FORMAT"`
    Syntax error in an LP file.

`"MSK_RES_ERR_WRITE_LP_FORMAT"`
    Problem cannot be written as an LP file.

`"MSK_RES_ERR_READ_LP_MISSING_END_TAG"`
    Syntax error in LP file. Possibly missing End tag.

`"MSK_RES_ERR_LP_FORMAT"`
    Syntax error in an LP file.

`"MSK_RES_ERR_WRITE_LP_NON_UNIQUE_NAME"`
    An auto-generated name is not unique.

`"MSK_RES_ERR_READ_LP_NONEXISTING_NAME"`
    A variable never occurred in objective or constraints.

`"MSK_RES_ERR_LP_WRITE_CONIC_PROBLEM"`
    The problem contains cones that cannot be written to an LP formatted file.

`"MSK_RES_ERR_LP_WRITE_GECO_PROBLEM"`
    The problem contains general convex terms that cannot be written to an LP formatted file.

`"MSK_RES_ERR_WRITING_FILE"`
    An error occurred while writing file

`"MSK_RES_ERR_OPF_FORMAT"`
    Syntax error in an OPF file

`"MSK_RES_ERR_OPF_NEW_VARIABLE"`
    Introducing new variables is now allowed. When a `[variables]` section is present, it is not allowed to introduce new variables later in the problem.

`"MSK_RES_ERR_INVALID_NAME_IN_SOL_FILE"`
    An invalid name occurred in a solution file.

`"MSK_RES_ERR_LP_INVALID_CON_NAME"`
    A constraint name is invalid when used in an LP formatted file.

`"MSK_RES_ERR_OPF_PREMATURE_EOF"`
    Premature end of file in an OPF file.

`"MSK_RES_ERR_JSON_SYNTAX"`
    Syntax error in an JSON data

`"MSK_RES_ERR_JSON_STRING"`
    Error in JSON string.

`"MSK_RES_ERR_JSON_NUMBER_OVERFLOW"`
    Invalid number entry - wrong type or value overflow.

`"MSK_RES_ERR_JSON_FORMAT"`
    Error in an JSON Task file

`"MSK_RES_ERR_JSON_DATA"`
    Inconsistent data in JSON Task file

`"MSK_RES_ERR_JSON_MISSING_DATA"`
    Missing data section in JSON task file.

`"MSK_RES_ERR_ARGUMENT_LENNEQ"`
    Incorrect length of arguments.

`"MSK_RES_ERR_ARGUMENT_TYPE"`
    Incorrect argument type.

`"MSK_RES_ERR_NR_ARGUMENTS"`
    Incorrect number of function arguments.

`"MSK_RES_ERR_IN_ARGUMENT"`
    A function argument is incorrect.

`"MSK_RES_ERR_ARGUMENT_DIMENSION"`
    A function argument is of incorrect dimension.

`"MSK_RES_ERR_INDEX_IS_TOO_SMALL"`
    An index in an argument is too small.

`"MSK_RES_ERR_INDEX_IS_TOO_LARGE"`
    An index in an argument is too large.

`"MSK_RES_ERR_PARAM_NAME"`
    The parameter name is not correct.

`"MSK_RES_ERR_PARAM_NAME_DOU"`
    The parameter name is not correct for a double parameter.

`"MSK_RES_ERR_PARAM_NAME_INT"`
    The parameter name is not correct for an integer parameter.

`"MSK_RES_ERR_PARAM_NAME_STR"`
    The parameter name is not correct for a string parameter.

`"MSK_RES_ERR_PARAM_INDEX"`
    Parameter index is out of range.

`"MSK_RES_ERR_PARAM_IS_TOO_LARGE"`
    The parameter value is too large.

`"MSK_RES_ERR_PARAM_IS_TOO_SMALL"`
    The parameter value is too small.

`"MSK_RES_ERR_PARAM_VALUE_STR"`
    The parameter value string is incorrect.

`"MSK_RES_ERR_PARAM_TYPE"`
    The parameter type is invalid.

`"MSK_RES_ERR_INF_DOU_INDEX"`
    A double information index is out of range for the specified type.

`"MSK_RES_ERR_INF_INT_INDEX"`
    An integer information index is out of range for the specified type.

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"MSK_RES_ERR_INDEX_ARR_IS_TOO_SMALL"
    An index in an array argument is too small.

"MSK_RES_ERR_INDEX_ARR_IS_TOO_LARGE"
    An index in an array argument is too large.

"MSK_RES_ERR_INF_LINT_INDEX"
    A long integer information index is out of range for the specified type.

"MSK_RES_ERR_ARG_IS_TOO_SMALL"
    The value of a argument is too small.

"MSK_RES_ERR_ARG_IS_TOO_LARGE"
    The value of a argument is too small.

"MSK_RES_ERR_INVALID_WHICHSOL"
    `whichsol` is invalid.

"MSK_RES_ERR_INF_DOU_NAME"
    A double information name is invalid.

"MSK_RES_ERR_INF_INT_NAME"
    An integer information name is invalid.

"MSK_RES_ERR_INF_TYPE"
    The information type is invalid.

"MSK_RES_ERR_INF_LINT_NAME"
    A long integer information name is invalid.

"MSK_RES_ERR_INDEX"
    An index is out of range.

"MSK_RES_ERR_WHICHSOL"
    The solution defined by `whichsol` does not exists.

"MSK_RES_ERR_SOLITEM"
    The solution item number `solitem` is invalid. Please note that *"MSK_SOL_ITEM_SNX"* is invalid for
    the basic solution.

"MSK_RES_ERR_WHICHITEM_NOT_ALLOWED"
    whichitem is unacceptable.

"MSK_RES_ERR_MAXNUMCON"
    The maximum number of constraints specified is smaller than the number of constraints in the
    task.

"MSK_RES_ERR_MAXNUMVAR"
    The maximum number of variables specified is smaller than the number of variables in the task.

"MSK_RES_ERR_MAXNUMBARVAR"
    The maximum number of semidefinite variables specified is smaller than the number of semidefinite
    variables in the task.

"MSK_RES_ERR_MAXNUMQNZ"
    The maximum number of non-zeros specified for the Q matrices is smaller than the number of
    non-zeros in the current Q matrices.

"MSK_RES_ERR_TOO_SMALL_MAX_NUM_NZ"
    The maximum number of non-zeros specified is too small.

"MSK_RES_ERR_INVALID_IDX"
    A specified index is invalid.

"MSK_RES_ERR_INVALID_MAX_NUM"
    A specified index is invalid.

"MSK_RES_ERR_NUMCONLIM"
    Maximum number of constraints limit is exceeded.

`"MSK_RES_ERR_NUMVARLIM"`
    Maximum number of variables limit is exceeded.

`"MSK_RES_ERR_TOO_SMALL_MAXNUMANZ"`
    The maximum number of non-zeros specified for A is smaller than the number of non-zeros in the current A.

`"MSK_RES_ERR_INV_APTRE"`
    `aptre[j]` is strictly smaller than `aptrb[j]` for some j.

`"MSK_RES_ERR_MUL_A_ELEMENT"`
    An element in A is defined multiple times.

`"MSK_RES_ERR_INV_BK"`
    Invalid bound key.

`"MSK_RES_ERR_INV_BKC"`
    Invalid bound key is specified for a constraint.

`"MSK_RES_ERR_INV_BKX"`
    An invalid bound key is specified for a variable.

`"MSK_RES_ERR_INV_VAR_TYPE"`
    An invalid variable type is specified for a variable.

`"MSK_RES_ERR_SOLVER_PROBTYPE"`
    Problem type does not match the chosen optimizer.

`"MSK_RES_ERR_OBJECTIVE_RANGE"`
    Empty objective range.

`"MSK_RES_ERR_FIRST"`
    Invalid `first`.

`"MSK_RES_ERR_LAST"`
    Invalid index `last`. A given index was out of expected range.

`"MSK_RES_ERR_NEGATIVE_SURPLUS"`
    Negative surplus.

`"MSK_RES_ERR_NEGATIVE_APPEND"`
    Cannot append a negative number.

`"MSK_RES_ERR_UNDEF_SOLUTION"`
    MOSEK has the following solution types:

  - an interior-point solution,

  - an basic solution,

  - and an integer solution.

    Each optimizer may set one or more of these solutions; e.g by default a successful optimization with the interior-point optimizer defines the interior-point solution, and, for linear problems, also the basic solution. This error occurs when asking for a solution or for information about a solution that is not defined.

`"MSK_RES_ERR_BASIS"`
    An invalid basis is specified. Either too many or too few basis variables are specified.

`"MSK_RES_ERR_INV_SKC"`
    Invalid value in `skc`.

`"MSK_RES_ERR_INV_SKX"`
    Invalid value in `skx`.

`"MSK_RES_ERR_INV_SKN"`
    Invalid value in `skn`.

`"MSK_RES_ERR_INV_SK_STR"`
　　Invalid status key string encountered.

`"MSK_RES_ERR_INV_SK"`
　　Invalid status key code.

`"MSK_RES_ERR_INV_CONE_TYPE_STR"`
　　Invalid cone type string encountered.

`"MSK_RES_ERR_INV_CONE_TYPE"`
　　Invalid cone type code is encountered.

`"MSK_RES_ERR_INVALID_SURPLUS"`
　　Invalid surplus.

`"MSK_RES_ERR_INV_NAME_ITEM"`
　　An invalid name item code is used.

`"MSK_RES_ERR_PRO_ITEM"`
　　An invalid problem is used.

`"MSK_RES_ERR_INVALID_FORMAT_TYPE"`
　　Invalid format type.

`"MSK_RES_ERR_FIRSTI"`
　　Invalid `firsti`.

`"MSK_RES_ERR_LASTI"`
　　Invalid `lasti`.

`"MSK_RES_ERR_FIRSTJ"`
　　Invalid `firstj`.

`"MSK_RES_ERR_LASTJ"`
　　Invalid `lastj`.

`"MSK_RES_ERR_MAX_LEN_IS_TOO_SMALL"`
　　An maximum length that is too small has been specified.

`"MSK_RES_ERR_NONLINEAR_EQUALITY"`
　　The model contains a nonlinear equality which defines a nonconvex set.

`"MSK_RES_ERR_NONCONVEX"`
　　The optimization problem is nonconvex.

`"MSK_RES_ERR_NONLINEAR_RANGED"`
　　Nonlinear constraints with finite lower and upper bound always define a nonconvex feasible set.

`"MSK_RES_ERR_CON_Q_NOT_PSD"`
　　The quadratic constraint matrix is not positive semidefinite as expected for a constraint with finite upper bound. This results in a nonconvex problem. The parameter `MSK_DPAR_CHECK_CONVEXITY_REL_TOL` can be used to relax the convexity check.

`"MSK_RES_ERR_CON_Q_NOT_NSD"`
　　The quadratic constraint matrix is not negative semidefinite as expected for a constraint with finite lower bound. This results in a nonconvex problem. The parameter `MSK_DPAR_CHECK_CONVEXITY_REL_TOL` can be used to relax the convexity check.

`"MSK_RES_ERR_OBJ_Q_NOT_PSD"`
　　The quadratic coefficient matrix in the objective is not positive semidefinite as expected for a minimization problem. The parameter `MSK_DPAR_CHECK_CONVEXITY_REL_TOL` can be used to relax the convexity check.

`"MSK_RES_ERR_OBJ_Q_NOT_NSD"`
　　The quadratic coefficient matrix in the objective is not negative semidefinite as expected for a maximization problem. The parameter `MSK_DPAR_CHECK_CONVEXITY_REL_TOL` can be used to relax the convexity check.

"MSK_RES_ERR_ARGUMENT_PERM_ARRAY"
    An invalid permutation array is specified.

"MSK_RES_ERR_CONE_INDEX"
    An index of a non-existing cone has been specified.

"MSK_RES_ERR_CONE_SIZE"
    A cone with too few members is specified.

"MSK_RES_ERR_CONE_OVERLAP"
    One or more of the variables in the cone to be added is already member of another cone. Now assume the variable is x_j then add a new variable say x_k and the constraint

$$x_j = x_k$$

    and then let x_k be member of the cone to be appended.

"MSK_RES_ERR_CONE_REP_VAR"
    A variable is included multiple times in the cone.

"MSK_RES_ERR_MAXNUMCONE"
    The value specified for maxnumcone is too small.

"MSK_RES_ERR_CONE_TYPE"
    Invalid cone type specified.

"MSK_RES_ERR_CONE_TYPE_STR"
    Invalid cone type specified.

"MSK_RES_ERR_CONE_OVERLAP_APPEND"
    The cone to be appended has one variable which is already member of another cone.

"MSK_RES_ERR_REMOVE_CONE_VARIABLE"
    A variable cannot be removed because it will make a cone invalid.

"MSK_RES_ERR_SOL_FILE_INVALID_NUMBER"
    An invalid number is specified in a solution file.

"MSK_RES_ERR_HUGE_C"
    A huge value in absolute size is specified for one c_j.

"MSK_RES_ERR_HUGE_AIJ"
    A numerically huge value is specified for an $a_{i,j}$ element in A. The parameter *MSK_DPAR_DATA_TOL_AIJ_HUGE* controls when an $a_{i,j}$ is considered huge.

"MSK_RES_ERR_DUPLICATE_AIJ"
    An element in the A matrix is specified twice.

"MSK_RES_ERR_LOWER_BOUND_IS_A_NAN"
    The lower bound specified is not a number (nan).

"MSK_RES_ERR_UPPER_BOUND_IS_A_NAN"
    The upper bound specified is not a number (nan).

"MSK_RES_ERR_INFINITE_BOUND"
    A numerically huge bound value is specified.

"MSK_RES_ERR_INV_QOBJ_SUBI"
    Invalid value in qosubi.

"MSK_RES_ERR_INV_QOBJ_SUBJ"
    Invalid value in qosubj.

"MSK_RES_ERR_INV_QOBJ_VAL"
    Invalid value in qoval.

"MSK_RES_ERR_INV_QCON_SUBK"
    Invalid value in qcsubk.

`"MSK_RES_ERR_INV_QCON_SUBI"`
>   Invalid value in `qcsubi`.

`"MSK_RES_ERR_INV_QCON_SUBJ"`
>   Invalid value in `qcsubj`.

`"MSK_RES_ERR_INV_QCON_VAL"`
>   Invalid value in `qcval`.

`"MSK_RES_ERR_QCON_SUBI_TOO_SMALL"`
>   Invalid value in `qcsubi`.

`"MSK_RES_ERR_QCON_SUBI_TOO_LARGE"`
>   Invalid value in `qcsubi`.

`"MSK_RES_ERR_QOBJ_UPPER_TRIANGLE"`
>   An element in the upper triangle of Q^o is specified. Only elements in the lower triangle should be specified.

`"MSK_RES_ERR_QCON_UPPER_TRIANGLE"`
>   An element in the upper triangle of a Q^k is specified. Only elements in the lower triangle should be specified.

`"MSK_RES_ERR_FIXED_BOUND_VALUES"`
>   A fixed constraint/variable has been specified using the bound keys but the numerical value of the lower and upper bound is different.

`"MSK_RES_ERR_NONLINEAR_FUNCTIONS_NOT_ALLOWED"`
>   An operation that is invalid for problems with nonlinear functions defined has been attempted.

`"MSK_RES_ERR_USER_FUNC_RET"`
>   An user function reported an error.

`"MSK_RES_ERR_USER_FUNC_RET_DATA"`
>   An user function returned invalid data.

`"MSK_RES_ERR_USER_NLO_FUNC"`
>   The user-defined nonlinear function reported an error.

`"MSK_RES_ERR_USER_NLO_EVAL"`
>   The user-defined nonlinear function reported an error.

`"MSK_RES_ERR_USER_NLO_EVAL_HESSUBI"`
>   The user-defined nonlinear function reported an invalid subscript in the Hessian.

`"MSK_RES_ERR_USER_NLO_EVAL_HESSUBJ"`
>   The user-defined nonlinear function reported an invalid subscript in the Hessian.

`"MSK_RES_ERR_INVALID_OBJECTIVE_SENSE"`
>   An invalid objective sense is specified.

`"MSK_RES_ERR_UNDEFINED_OBJECTIVE_SENSE"`
>   The objective sense has not been specified before the optimization.

`"MSK_RES_ERR_Y_IS_UNDEFINED"`
>   The solution item y is undefined.

`"MSK_RES_ERR_NAN_IN_DOUBLE_DATA"`
>   An invalid floating point value was used in some double data.

`"MSK_RES_ERR_NAN_IN_BLC"`
> l^c contains an invalid floating point value, i.e. a `NaN`.

`"MSK_RES_ERR_NAN_IN_BUC"`
> u^c contains an invalid floating point value, i.e. a `NaN`.

`"MSK_RES_ERR_NAN_IN_C"`
> c contains an invalid floating point value, i.e. a `NaN`.

`"MSK_RES_ERR_NAN_IN_BLX"`
> l^x contains an invalid floating point value, i.e. a `NaN`.

`"MSK_RES_ERR_NAN_IN_BUX"`
> u^x contains an invalid floating point value, i.e. a `NaN`.

`"MSK_RES_ERR_INVALID_AIJ"`
> $a_{i,j}$ contains an invalid floating point value, i.e. a `NaN` or an infinite value.

`"MSK_RES_ERR_SYM_MAT_INVALID"`
> A symmetric matrix contains an invalid floating point value, i.e. a `NaN` or an infinite value.

`"MSK_RES_ERR_SYM_MAT_HUGE"`
> A symmetric matrix contains a huge value in absolute size. The parameter
> *MSK_DPAR_DATA_SYM_MAT_TOL_HUGE* controls when an $e_{i,j}$ is considered huge.

`"MSK_RES_ERR_INV_PROBLEM"`
> Invalid problem type. Probably a nonconvex problem has been specified.

`"MSK_RES_ERR_MIXED_CONIC_AND_NL"`
> The problem contains nonlinear terms conic constraints. The requested operation cannot be applied
> to this type of problem.

`"MSK_RES_ERR_GLOBAL_INV_CONIC_PROBLEM"`
> The global optimizer can only be applied to problems without semidefinite variables.

`"MSK_RES_ERR_INV_OPTIMIZER"`
> An invalid optimizer has been chosen for the problem. This means that the simplex or the conic
> optimizer is chosen to optimize a nonlinear problem.

`"MSK_RES_ERR_MIO_NO_OPTIMIZER"`
> No optimizer is available for the current class of integer optimization problems.

`"MSK_RES_ERR_NO_OPTIMIZER_VAR_TYPE"`
> No optimizer is available for this class of optimization problems.

`"MSK_RES_ERR_FINAL_SOLUTION"`
> An error occurred during the solution finalization.

`"MSK_RES_ERR_POSTSOLVE"`
> An error occurred during the postsolve. Please contact **MOSEK** support.

`"MSK_RES_ERR_OVERFLOW"`
> A computation produced an overflow i.e. a very large number.

`"MSK_RES_ERR_NO_BASIS_SOL"`
> No basic solution is defined.

`"MSK_RES_ERR_BASIS_FACTOR"`
> The factorization of the basis is invalid.

`"MSK_RES_ERR_BASIS_SINGULAR"`
> The basis is singular and hence cannot be factored.

`"MSK_RES_ERR_FACTOR"`
> An error occurred while factorizing a matrix.

`"MSK_RES_ERR_FEASREPAIR_CANNOT_RELAX"`
> An optimization problem cannot be relaxed. This is the case e.g. for general nonlinear optimization
> problems.

`"MSK_RES_ERR_FEASREPAIR_SOLVING_RELAXED"`
> The relaxed problem could not be solved to optimality. Please consult the log file for further details.

`"MSK_RES_ERR_FEASREPAIR_INCONSISTENT_BOUND"`
> The upper bound is less than the lower bound for a variable or a constraint. Please correct this
> before running the feasibility repair.

"MSK_RES_ERR_REPAIR_INVALID_PROBLEM"
    The feasibility repair does not support the specified problem type.

"MSK_RES_ERR_REPAIR_OPTIMIZATION_FAILED"
    Computation the optimal relaxation failed. The cause may have been numerical problems.

"MSK_RES_ERR_NAME_MAX_LEN"
    A name is longer than the buffer that is supposed to hold it.

"MSK_RES_ERR_NAME_IS_NULL"
    The name buffer is a NULL pointer.

"MSK_RES_ERR_INVALID_COMPRESSION"
    Invalid compression type.

"MSK_RES_ERR_INVALID_IOMODE"
    Invalid io mode.

"MSK_RES_ERR_NO_PRIMAL_INFEAS_CER"
    A certificate of primal infeasibility is not available.

"MSK_RES_ERR_NO_DUAL_INFEAS_CER"
    A certificate of infeasibility is not available.

"MSK_RES_ERR_NO_SOLUTION_IN_CALLBACK"
    The required solution is not available.

"MSK_RES_ERR_INV_MARKI"
    Invalid value in marki.

"MSK_RES_ERR_INV_MARKJ"
    Invalid value in markj.

"MSK_RES_ERR_INV_NUMI"
    Invalid numi.

"MSK_RES_ERR_INV_NUMJ"
    Invalid numj.

"MSK_RES_ERR_CANNOT_CLONE_NL"
    A task with a nonlinear function callback cannot be cloned.

"MSK_RES_ERR_CANNOT_HANDLE_NL"
    A function cannot handle a task with nonlinear function callbacks.

"MSK_RES_ERR_INVALID_ACCMODE"
    An invalid access mode is specified.

"MSK_RES_ERR_TASK_INCOMPATIBLE"
    The Task file is incompatible with this platform. This results from reading a file on a 32 bit platform generated on a 64 bit platform.

"MSK_RES_ERR_TASK_INVALID"
    The Task file is invalid.

"MSK_RES_ERR_TASK_WRITE"
    Failed to write the task file.

"MSK_RES_ERR_LU_MAX_NUM_TRIES"
    Could not compute the LU factors of the matrix within the maximum number of allowed tries.

"MSK_RES_ERR_INVALID_UTF8"
    An invalid UTF8 string is encountered.

"MSK_RES_ERR_INVALID_WCHAR"
    An invalid `wchar` string is encountered.

"MSK_RES_ERR_NO_DUAL_FOR_ITG_SOL"
    No dual information is available for the integer solution.

`"MSK_RES_ERR_NO_SNX_FOR_BAS_SOL"`
 s_n^x is not available for the basis solution.

`"MSK_RES_ERR_INTERNAL"`
 An internal error occurred. Please report this problem.

`"MSK_RES_ERR_API_ARRAY_TOO_SMALL"`
 An input array was too short.

`"MSK_RES_ERR_API_CB_CONNECT"`
 Failed to connect a callback object.

`"MSK_RES_ERR_API_FATAL_ERROR"`
 An internal error occurred in the API. Please report this problem.

`"MSK_RES_ERR_API_INTERNAL"`
 An internal fatal error occurred in an interface function.

`"MSK_RES_ERR_SEN_FORMAT"`
 Syntax error in sensitivity analysis file.

`"MSK_RES_ERR_SEN_UNDEF_NAME"`
 An undefined name was encountered in the sensitivity analysis file.

`"MSK_RES_ERR_SEN_INDEX_RANGE"`
 Index out of range in the sensitivity analysis file.

`"MSK_RES_ERR_SEN_BOUND_INVALID_UP"`
 Analysis of upper bound requested for an index, where no upper bound exists.

`"MSK_RES_ERR_SEN_BOUND_INVALID_LO"`
 Analysis of lower bound requested for an index, where no lower bound exists.

`"MSK_RES_ERR_SEN_INDEX_INVALID"`
 Invalid range given in the sensitivity file.

`"MSK_RES_ERR_SEN_INVALID_REGEXP"`
 Syntax error in regexp or regexp longer than 1024.

`"MSK_RES_ERR_SEN_SOLUTION_STATUS"`
 No optimal solution found to the original problem given for sensitivity analysis.

`"MSK_RES_ERR_SEN_NUMERICAL"`
 Numerical difficulties encountered performing the sensitivity analysis.

`"MSK_RES_ERR_SEN_UNHANDLED_PROBLEM_TYPE"`
 Sensitivity analysis cannot be performed for the specified problem. Sensitivity analysis is only possible for linear problems.

`"MSK_RES_ERR_UNB_STEP_SIZE"`
 A step size in an optimizer was unexpectedly unbounded. For instance, if the step-size becomes unbounded in phase 1 of the simplex algorithm then an error occurs. Normally this will happen only if the problem is badly formulated. Please contact **MOSEK** support if this error occurs.

`"MSK_RES_ERR_IDENTICAL_TASKS"`
 Some tasks related to this function call were identical. Unique tasks were expected.

`"MSK_RES_ERR_AD_INVALID_CODELIST"`
 The code list data was invalid.

`"MSK_RES_ERR_INTERNAL_TEST_FAILED"`
 An internal unit test function failed.

`"MSK_RES_ERR_XML_INVALID_PROBLEM_TYPE"`
 The problem type is not supported by the XML format.

`"MSK_RES_ERR_INVALID_AMPL_STUB"`
 Invalid AMPL stub.

`"MSK_RES_ERR_INT64_TO_INT32_CAST"`
> An 32 bit integer could not cast to a 64 bit integer.

`"MSK_RES_ERR_SIZE_LICENSE_NUMCORES"`
> The computer contains more cpu cores than the license allows for.

`"MSK_RES_ERR_INFEAS_UNDEFINED"`
> The requested value is not defined for this solution type.

`"MSK_RES_ERR_NO_BARX_FOR_SOLUTION"`
> There is no $\overline{X}$ available for the solution specified. In particular note there are no $\overline{X}$ defined for the basic and integer solutions.

`"MSK_RES_ERR_NO_BARS_FOR_SOLUTION"`
> There is no $\bar{s}$ available for the solution specified. In particular note there are no $\bar{s}$ defined for the basic and integer solutions.

`"MSK_RES_ERR_BAR_VAR_DIM"`
> The dimension of a symmetric matrix variable has to greater than 0.

`"MSK_RES_ERR_SYM_MAT_INVALID_ROW_INDEX"`
> A row index specified for sparse symmetric matrix is invalid.

`"MSK_RES_ERR_SYM_MAT_INVALID_COL_INDEX"`
> A column index specified for sparse symmetric matrix is invalid.

`"MSK_RES_ERR_SYM_MAT_NOT_LOWER_TRINGULAR"`
> Only the lower triangular part of sparse symmetric matrix should be specified.

`"MSK_RES_ERR_SYM_MAT_INVALID_VALUE"`
> The numerical value specified in a sparse symmetric matrix is not a value floating value.

`"MSK_RES_ERR_SYM_MAT_DUPLICATE"`
> A value in a symmetric matric as been specified more than once.

`"MSK_RES_ERR_INVALID_SYM_MAT_DIM"`
> A sparse symmetric matrix of invalid dimension is specified.

`"MSK_RES_ERR_INVALID_FILE_FORMAT_FOR_SYM_MAT"`
> The file format does not support a problem with symmetric matrix variables.

`"MSK_RES_ERR_INVALID_FILE_FORMAT_FOR_CONES"`
> The file format does not support a problem with conic constraints.

`"MSK_RES_ERR_INVALID_FILE_FORMAT_FOR_GENERAL_NL"`
> The file format does not support a problem with general nonlinear terms.

`"MSK_RES_ERR_DUPLICATE_CONSTRAINT_NAMES"`
> Two constraint names are identical.

`"MSK_RES_ERR_DUPLICATE_VARIABLE_NAMES"`
> Two variable names are identical.

`"MSK_RES_ERR_DUPLICATE_BARVARIABLE_NAMES"`
> Two barvariable names are identical.

`"MSK_RES_ERR_DUPLICATE_CONE_NAMES"`
> Two cone names are identical.

`"MSK_RES_ERR_NON_UNIQUE_ARRAY"`
> An array does not contain unique elements.

`"MSK_RES_ERR_ARGUMENT_IS_TOO_LARGE"`
> The value of a function argument is too large.

`"MSK_RES_ERR_MIO_INTERNAL"`
> A fatal error occurred in the mixed integer optimizer. Please contact **MOSEK** support.

`"MSK_RES_ERR_INVALID_PROBLEM_TYPE"`
    An invalid problem type.

`"MSK_RES_ERR_UNHANDLED_SOLUTION_STATUS"`
    Unhandled solution status.

`"MSK_RES_ERR_UPPER_TRIANGLE"`
    An element in the upper triangle of a lower triangular matrix is specified.

`"MSK_RES_ERR_LAU_SINGULAR_MATRIX"`
    A matrix is singular.

`"MSK_RES_ERR_LAU_NOT_POSITIVE_DEFINITE"`
    A matrix is not positive definite.

`"MSK_RES_ERR_LAU_INVALID_LOWER_TRIANGULAR_MATRIX"`
    An invalid lower triangular matrix.

`"MSK_RES_ERR_LAU_UNKNOWN"`
    An unknown error.

`"MSK_RES_ERR_LAU_ARG_M"`
    Invalid argument m.

`"MSK_RES_ERR_LAU_ARG_N"`
    Invalid argument n.

`"MSK_RES_ERR_LAU_ARG_K"`
    Invalid argument k.

`"MSK_RES_ERR_LAU_ARG_TRANSA"`
    Invalid argument transa.

`"MSK_RES_ERR_LAU_ARG_TRANSB"`
    Invalid argument transb.

`"MSK_RES_ERR_LAU_ARG_UPLO"`
    Invalid argument uplo.

`"MSK_RES_ERR_LAU_ARG_TRANS"`
    Invalid argument trans.

`"MSK_RES_ERR_LAU_INVALID_SPARSE_SYMMETRIC_MATRIX"`
    An invalid sparse symmetric matrix is specfified. Note only the lower triangular part with no
    duplicates is specifed.

`"MSK_RES_ERR_CBF_PARSE"`
    An error occurred while parsing an CBF file.

`"MSK_RES_ERR_CBF_OBJ_SENSE"`
    An invalid objective sense is specified.

`"MSK_RES_ERR_CBF_NO_VARIABLES"`
    No variables are specified.

`"MSK_RES_ERR_CBF_TOO_MANY_CONSTRAINTS"`
    Too many constraints specified.

`"MSK_RES_ERR_CBF_TOO_MANY_VARIABLES"`
    Too many variables specified.

`"MSK_RES_ERR_CBF_NO_VERSION_SPECIFIED"`
    No version specified.

`"MSK_RES_ERR_CBF_SYNTAX"`
    Invalid syntax.

`"MSK_RES_ERR_CBF_DUPLICATE_OBJ"`
    Duplicate OBJ keyword.

12.4. Response codes

`"MSK_RES_ERR_CBF_DUPLICATE_CON"`
    Duplicate CON keyword.

`"MSK_RES_ERR_CBF_DUPLICATE_VAR"`
    Duplicate VAR keyword.

`"MSK_RES_ERR_CBF_DUPLICATE_INT"`
    Duplicate INT keyword.

`"MSK_RES_ERR_CBF_INVALID_VAR_TYPE"`
    Invalid variable type.

`"MSK_RES_ERR_CBF_INVALID_CON_TYPE"`
    Invalid constraint type.

`"MSK_RES_ERR_CBF_INVALID_DOMAIN_DIMENSION"`
    Invalid domain dimension.

`"MSK_RES_ERR_CBF_DUPLICATE_OBJACOORD"`
    Duplicate index in OBJCOORD.

`"MSK_RES_ERR_CBF_DUPLICATE_BCOORD"`
    Duplicate index in BCOORD.

`"MSK_RES_ERR_CBF_DUPLICATE_ACOORD"`
    Duplicate index in ACOORD.

`"MSK_RES_ERR_CBF_TOO_FEW_VARIABLES"`
    Too few variables defined.

`"MSK_RES_ERR_CBF_TOO_FEW_CONSTRAINTS"`
    Too few constraints defined.

`"MSK_RES_ERR_CBF_TOO_FEW_INTS"`
    Too few ints are specified.

`"MSK_RES_ERR_CBF_TOO_MANY_INTS"`
    Too many ints are specified.

`"MSK_RES_ERR_CBF_INVALID_INT_INDEX"`
    Invalid INT index.

`"MSK_RES_ERR_CBF_UNSUPPORTED"`
    Unsupported feature is present.

`"MSK_RES_ERR_CBF_DUPLICATE_PSDVAR"`
    Duplicate PSDVAR keyword.

`"MSK_RES_ERR_CBF_INVALID_PSDVAR_DIMENSION"`
    Invalid PSDVAR dimmension.

`"MSK_RES_ERR_CBF_TOO_FEW_PSDVAR"`
    Too few variables defined.

`"MSK_RES_ERR_MIO_INVALID_ROOT_OPTIMIZER"`
    An invalid root optimizer was selected for the problem type.

`"MSK_RES_ERR_MIO_INVALID_NODE_OPTIMIZER"`
    An invalid node optimizer was selected for the problem type.

`"MSK_RES_ERR_TOCONIC_CONSTR_Q_NOT_PSD"`
    The matrix defining the quadratric part of constraint is not positive semidefinite.

`"MSK_RES_ERR_TOCONIC_CONSTRAINT_FX"`
    The quadratic constraint is an equality, thus not convex.

`"MSK_RES_ERR_TOCONIC_CONSTRAINT_RA"`
    The quadratic constraint has finite lower and upper bound, and therefore it is not convex.

`"MSK_RES_ERR_TOCONIC_CONSTR_NOT_CONIC"`
> The constraint is not conic representable.

`"MSK_RES_ERR_TOCONIC_OBJECTIVE_NOT_PSD"`
> The matrix defining the quadratic part of the objective function is not positive semidefinite.

`"MSK_RES_ERR_SERVER_CONNECT"`
> Failed to connect to remote solver server. The server string or the port string were invalid, or the server did not accept connection.

`"MSK_RES_ERR_SERVER_PROTOCOL"`
> Unexpected message or data from solver server.

`"MSK_RES_ERR_SERVER_STATUS"`
> Server returned non-ok HTTP status code

`"MSK_RES_ERR_SERVER_TOKEN"`
> The job ID specified is incorrect or invalid

12.5 Enumerations

`language`
> Language selection constants
>
> `"MSK_LANG_ENG"`
>> English language selection
>
> `"MSK_LANG_DAN"`
>> Danish language selection

`accmode`
> Constraint or variable access modes
>
> `"MSK_ACC_VAR"`
>> Access data by columns (variable oriented)
>
> `"MSK_ACC_CON"`
>> Access data by rows (constraint oriented)

`basindtype`
> Basis identification
>
> `"MSK_BI_NEVER"`
>> Never do basis identification.
>
> `"MSK_BI_ALWAYS"`
>> Basis identification is always performed even if the interior-point optimizer terminates abnormally.
>
> `"MSK_BI_NO_ERROR"`
>> Basis identification is performed if the interior-point optimizer terminates without an error.
>
> `"MSK_BI_IF_FEASIBLE"`
>> Basis identification is not performed if the interior-point optimizer terminates with a problem status saying that the problem is primal or dual infeasible.
>
> `"MSK_BI_RESERVERED"`
>> Not currently in use.

`boundkey`
> Bound keys
>
> `"MSK_BK_LO"`
>> The constraint or variable has a finite lower bound and an infinite upper bound.

"MSK_BK_UP"
> The constraint or variable has an infinite lower bound and an finite upper bound.

"MSK_BK_FX"
> The constraint or variable is fixed.

"MSK_BK_FR"
> The constraint or variable is free.

"MSK_BK_RA"
> The constraint or variable is ranged.

mark
> Mark

"MSK_MARK_LO"
> The lower bound is selected for sensitivity analysis.

"MSK_MARK_UP"
> The upper bound is selected for sensitivity analysis.

simdegen
> Degeneracy strategies

"MSK_SIM_DEGEN_NONE"
> The simplex optimizer should use no degeneration strategy.

"MSK_SIM_DEGEN_FREE"
> The simplex optimizer chooses the degeneration strategy.

"MSK_SIM_DEGEN_AGGRESSIVE"
> The simplex optimizer should use an aggressive degeneration strategy.

"MSK_SIM_DEGEN_MODERATE"
> The simplex optimizer should use a moderate degeneration strategy.

"MSK_SIM_DEGEN_MINIMUM"
> The simplex optimizer should use a minimum degeneration strategy.

transpose
> Transposed matrix.

"MSK_TRANSPOSE_NO"
> No transpose is applied.

"MSK_TRANSPOSE_YES"
> A transpose is applied.

uplo
> Triangular part of a symmetric matrix.

"MSK_UPLO_LO"
> Lower part.

"MSK_UPLO_UP"
> Upper part

simreform
> Problem reformulation.

"MSK_SIM_REFORMULATION_ON"
> Allow the simplex optimizer to reformulate the problem.

"MSK_SIM_REFORMULATION_OFF"
> Disallow the simplex optimizer to reformulate the problem.

"MSK_SIM_REFORMULATION_FREE"
> The simplex optimizer can choose freely.

> "MSK_SIM_REFORMULATION_AGGRESSIVE"
>> The simplex optimizer should use an aggressive reformulation strategy.

simdupvec
> Exploit duplicate columns.

> "MSK_SIM_EXPLOIT_DUPVEC_ON"
>> Allow the simplex optimizer to exploit duplicated columns.

> "MSK_SIM_EXPLOIT_DUPVEC_OFF"
>> Disallow the simplex optimizer to exploit duplicated columns.

> "MSK_SIM_EXPLOIT_DUPVEC_FREE"
>> The simplex optimizer can choose freely.

simhotstart
> Hot-start type employed by the simplex optimizer

> "MSK_SIM_HOTSTART_NONE"
>> The simplex optimizer performs a coldstart.

> "MSK_SIM_HOTSTART_FREE"
>> The simplex optimize chooses the hot-start type.

> "MSK_SIM_HOTSTART_STATUS_KEYS"
>> Only the status keys of the constraints and variables are used to choose the type of hot-start.

intpnthotstart
> Hot-start type employed by the interior-point optimizers.

> "MSK_INTPNT_HOTSTART_NONE"
>> The interior-point optimizer performs a coldstart.

> "MSK_INTPNT_HOTSTART_PRIMAL"
>> The interior-point optimizer exploits the primal solution only.

> "MSK_INTPNT_HOTSTART_DUAL"
>> The interior-point optimizer exploits the dual solution only.

> "MSK_INTPNT_HOTSTART_PRIMAL_DUAL"
>> The interior-point optimizer exploits both the primal and dual solution.

callbackcode
> Progress callback codes

> "MSK_CALLBACK_BEGIN_BI"
>> The basis identification procedure has been started.

> "MSK_CALLBACK_BEGIN_CONIC"
>> The callback function is called when the conic optimizer is started.

> "MSK_CALLBACK_BEGIN_DUAL_BI"
>> The callback function is called from within the basis identification procedure when the dual phase is started.

> "MSK_CALLBACK_BEGIN_DUAL_SENSITIVITY"
>> Dual sensitivity analysis is started.

> "MSK_CALLBACK_BEGIN_DUAL_SETUP_BI"
>> The callback function is called when the dual BI phase is started.

> "MSK_CALLBACK_BEGIN_DUAL_SIMPLEX"
>> The callback function is called when the dual simplex optimizer started.

> "MSK_CALLBACK_BEGIN_DUAL_SIMPLEX_BI"
>> The callback function is called from within the basis identification procedure when the dual simplex clean-up phase is started.

"MSK_CALLBACK_BEGIN_FULL_CONVEXITY_CHECK"
    Begin full convexity check.

"MSK_CALLBACK_BEGIN_INFEAS_ANA"
    The callback function is called when the infeasibility analyzer is started.

"MSK_CALLBACK_BEGIN_INTPNT"
    The callback function is called when the interior-point optimizer is started.

"MSK_CALLBACK_BEGIN_LICENSE_WAIT"
    Begin waiting for license.

"MSK_CALLBACK_BEGIN_MIO"
    The callback function is called when the mixed-integer optimizer is started.

"MSK_CALLBACK_BEGIN_OPTIMIZER"
    The callback function is called when the optimizer is started.

"MSK_CALLBACK_BEGIN_PRESOLVE"
    The callback function is called when the presolve is started.

"MSK_CALLBACK_BEGIN_PRIMAL_BI"
    The callback function is called from within the basis identification procedure when the primal
    phase is started.

"MSK_CALLBACK_BEGIN_PRIMAL_REPAIR"
    Begin primal feasibility repair.

"MSK_CALLBACK_BEGIN_PRIMAL_SENSITIVITY"
    Primal sensitivity analysis is started.

"MSK_CALLBACK_BEGIN_PRIMAL_SETUP_BI"
    The callback function is called when the primal BI setup is started.

"MSK_CALLBACK_BEGIN_PRIMAL_SIMPLEX"
    The callback function is called when the primal simplex optimizer is started.

"MSK_CALLBACK_BEGIN_PRIMAL_SIMPLEX_BI"
    The callback function is called from within the basis identification procedure when the primal
    simplex clean-up phase is started.

"MSK_CALLBACK_BEGIN_QCQO_REFORMULATE"
    Begin QCQO reformulation.

"MSK_CALLBACK_BEGIN_READ"
    MOSEK has started reading a problem file.

"MSK_CALLBACK_BEGIN_ROOT_CUTGEN"
    The callback function is called when root cut generation is started.

"MSK_CALLBACK_BEGIN_SIMPLEX"
    The callback function is called when the simplex optimizer is started.

"MSK_CALLBACK_BEGIN_SIMPLEX_BI"
    The callback function is called from within the basis identification procedure when the simplex
    clean-up phase is started.

"MSK_CALLBACK_BEGIN_TO_CONIC"
    Begin conic reformulation.

"MSK_CALLBACK_BEGIN_WRITE"
    MOSEK has started writing a problem file.

"MSK_CALLBACK_CONIC"
    The callback function is called from within the conic optimizer after the information database
    has been updated.

"MSK_CALLBACK_DUAL_SIMPLEX"
    The callback function is called from within the dual simplex optimizer.

`"MSK_CALLBACK_END_BI"`
> The callback function is called when the basis identification procedure is terminated.

`"MSK_CALLBACK_END_CONIC"`
> The callback function is called when the conic optimizer is terminated.

`"MSK_CALLBACK_END_DUAL_BI"`
> The callback function is called from within the basis identification procedure when the dual phase is terminated.

`"MSK_CALLBACK_END_DUAL_SENSITIVITY"`
> Dual sensitivity analysis is terminated.

`"MSK_CALLBACK_END_DUAL_SETUP_BI"`
> The callback function is called when the dual BI phase is terminated.

`"MSK_CALLBACK_END_DUAL_SIMPLEX"`
> The callback function is called when the dual simplex optimizer is terminated.

`"MSK_CALLBACK_END_DUAL_SIMPLEX_BI"`
> The callback function is called from within the basis identification procedure when the dual clean-up phase is terminated.

`"MSK_CALLBACK_END_FULL_CONVEXITY_CHECK"`
> End full convexity check.

`"MSK_CALLBACK_END_INFEAS_ANA"`
> The callback function is called when the infeasibility analyzer is terminated.

`"MSK_CALLBACK_END_INTPNT"`
> The callback function is called when the interior-point optimizer is terminated.

`"MSK_CALLBACK_END_LICENSE_WAIT"`
> End waiting for license.

`"MSK_CALLBACK_END_MIO"`
> The callback function is called when the mixed-integer optimizer is terminated.

`"MSK_CALLBACK_END_OPTIMIZER"`
> The callback function is called when the optimizer is terminated.

`"MSK_CALLBACK_END_PRESOLVE"`
> The callback function is called when the presolve is completed.

`"MSK_CALLBACK_END_PRIMAL_BI"`
> The callback function is called from within the basis identification procedure when the primal phase is terminated.

`"MSK_CALLBACK_END_PRIMAL_REPAIR"`
> End primal feasibility repair.

`"MSK_CALLBACK_END_PRIMAL_SENSITIVITY"`
> Primal sensitivity analysis is terminated.

`"MSK_CALLBACK_END_PRIMAL_SETUP_BI"`
> The callback function is called when the primal BI setup is terminated.

`"MSK_CALLBACK_END_PRIMAL_SIMPLEX"`
> The callback function is called when the primal simplex optimizer is terminated.

`"MSK_CALLBACK_END_PRIMAL_SIMPLEX_BI"`
> The callback function is called from within the basis identification procedure when the primal clean-up phase is terminated.

`"MSK_CALLBACK_END_QCQO_REFORMULATE"`
> End QCQO reformulation.

`"MSK_CALLBACK_END_READ"`
> **MOSEK** has finished reading a problem file.

"MSK_CALLBACK_END_ROOT_CUTGEN"
> The callback function is called when root cut generation is is terminated.

"MSK_CALLBACK_END_SIMPLEX"
> The callback function is called when the simplex optimizer is terminated.

"MSK_CALLBACK_END_SIMPLEX_BI"
> The callback function is called from within the basis identification procedure when the simplex clean-up phase is terminated.

"MSK_CALLBACK_END_TO_CONIC"
> End conic reformulation.

"MSK_CALLBACK_END_WRITE"
> **MOSEK** has finished writing a problem file.

"MSK_CALLBACK_IM_BI"
> The callback function is called from within the basis identification procedure at an intermediate point.

"MSK_CALLBACK_IM_CONIC"
> The callback function is called at an intermediate stage within the conic optimizer where the information database has not been updated.

"MSK_CALLBACK_IM_DUAL_BI"
> The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

"MSK_CALLBACK_IM_DUAL_SENSIVITY"
> The callback function is called at an intermediate stage of the dual sensitivity analysis.

"MSK_CALLBACK_IM_DUAL_SIMPLEX"
> The callback function is called at an intermediate point in the dual simplex optimizer.

"MSK_CALLBACK_IM_FULL_CONVEXITY_CHECK"
> The callback function is called at an intermediate stage of the full convexity check.

"MSK_CALLBACK_IM_INTPNT"
> The callback function is called at an intermediate stage within the interior-point optimizer where the information database has not been updated.

"MSK_CALLBACK_IM_LICENSE_WAIT"
> **MOSEK** is waiting for a license.

"MSK_CALLBACK_IM_LU"
> The callback function is called from within the LU factorization procedure at an intermediate point.

"MSK_CALLBACK_IM_MIO"
> The callback function is called at an intermediate point in the mixed-integer optimizer.

"MSK_CALLBACK_IM_MIO_DUAL_SIMPLEX"
> The callback function is called at an intermediate point in the mixed-integer optimizer while running the dual simplex optimizer.

"MSK_CALLBACK_IM_MIO_INTPNT"
> The callback function is called at an intermediate point in the mixed-integer optimizer while running the interior-point optimizer.

"MSK_CALLBACK_IM_MIO_PRIMAL_SIMPLEX"
> The callback function is called at an intermediate point in the mixed-integer optimizer while running the primal simplex optimizer.

"MSK_CALLBACK_IM_ORDER"
> The callback function is called from within the matrix ordering procedure at an intermediate point.

"MSK_CALLBACK_IM_PRESOLVE"
The callback function is called from within the presolve procedure at an intermediate stage.

"MSK_CALLBACK_IM_PRIMAL_BI"
The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

"MSK_CALLBACK_IM_PRIMAL_SENSIVITY"
The callback function is called at an intermediate stage of the primal sensitivity analysis.

"MSK_CALLBACK_IM_PRIMAL_SIMPLEX"
The callback function is called at an intermediate point in the primal simplex optimizer.

"MSK_CALLBACK_IM_QO_REFORMULATE"
The callback function is called at an intermediate stage of the conic quadratic reformulation.

"MSK_CALLBACK_IM_READ"
Intermediate stage in reading.

"MSK_CALLBACK_IM_ROOT_CUTGEN"
The callback is called from within root cut generation at an intermediate stage.

"MSK_CALLBACK_IM_SIMPLEX"
The callback function is called from within the simplex optimizer at an intermediate point.

"MSK_CALLBACK_IM_SIMPLEX_BI"
The callback function is called from within the basis identification procedure at an intermediate point in the simplex clean-up phase. The frequency of the callbacks is controlled by the *MSK_IPAR_LOG_SIM_FREQ* parameter.

"MSK_CALLBACK_INTPNT"
The callback function is called from within the interior-point optimizer after the information database has been updated.

"MSK_CALLBACK_NEW_INT_MIO"
The callback function is called after a new integer solution has been located by the mixed-integer optimizer.

"MSK_CALLBACK_PRIMAL_SIMPLEX"
The callback function is called from within the primal simplex optimizer.

"MSK_CALLBACK_READ_OPF"
The callback function is called from the OPF reader.

"MSK_CALLBACK_READ_OPF_SECTION"
A chunk of Q non-zeros has been read from a problem file.

"MSK_CALLBACK_SOLVING_REMOTE"
The callback function is called while the task is being solved on a remote server.

"MSK_CALLBACK_UPDATE_DUAL_BI"
The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

"MSK_CALLBACK_UPDATE_DUAL_SIMPLEX"
The callback function is called in the dual simplex optimizer.

"MSK_CALLBACK_UPDATE_DUAL_SIMPLEX_BI"
The callback function is called from within the basis identification procedure at an intermediate point in the dual simplex clean-up phase. The frequency of the callbacks is controlled by the *MSK_IPAR_LOG_SIM_FREQ* parameter.

"MSK_CALLBACK_UPDATE_PRESOLVE"
The callback function is called from within the presolve procedure.

"MSK_CALLBACK_UPDATE_PRIMAL_BI"
The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

"MSK_CALLBACK_UPDATE_PRIMAL_SIMPLEX"
>   The callback function is called in the primal simplex optimizer.

"MSK_CALLBACK_UPDATE_PRIMAL_SIMPLEX_BI"
>   The callback function is called from within the basis identification procedure at an intermediate point in the primal simplex clean-up phase. The frequency of the callbacks is controlled by the *MSK_IPAR_LOG_SIM_FREQ* parameter.

"MSK_CALLBACK_WRITE_OPF"
>   The callback function is called from the OPF writer.

checkconvexitytype
>   Types of convexity checks.

"MSK_CHECK_CONVEXITY_NONE"
>   No convexity check.

"MSK_CHECK_CONVEXITY_SIMPLE"
>   Perform simple and fast convexity check.

"MSK_CHECK_CONVEXITY_FULL"
>   Perform a full convexity check.

compresstype
>   Compression types

"MSK_COMPRESS_NONE"
>   No compression is used.

"MSK_COMPRESS_FREE"
>   The type of compression used is chosen automatically.

"MSK_COMPRESS_GZIP"
>   The type of compression used is gzip compatible.

conetype
>   Cone types

"MSK_CT_QUAD"
>   The cone is a quadratic cone.

"MSK_CT_RQUAD"
>   The cone is a rotated quadratic cone.

nametype
>   Name types

"MSK_NAME_TYPE_GEN"
>   General names. However, no duplicate and blank names are allowed.

"MSK_NAME_TYPE_MPS"
>   MPS type names.

"MSK_NAME_TYPE_LP"
>   LP type names.

symmattype
>   Cone types

"MSK_SYMMAT_TYPE_SPARSE"
>   Sparse symmetric matrix.

dataformat
>   Data format types

"MSK_DATA_FORMAT_EXTENSION"
>   The file extension is used to determine the data file format.

"MSK_DATA_FORMAT_MPS"
>   The data file is MPS formatted.

"MSK_DATA_FORMAT_LP"
>   The data file is LP formatted.

"MSK_DATA_FORMAT_OP"
>   The data file is an optimization problem formatted file.

"MSK_DATA_FORMAT_XML"
>   The data file is an XML formatted file.

"MSK_DATA_FORMAT_FREE_MPS"
>   The data a free MPS formatted file.

"MSK_DATA_FORMAT_TASK"
>   Generic task dump file.

"MSK_DATA_FORMAT_CB"
>   Conic benchmark format,

"MSK_DATA_FORMAT_JSON_TASK"
>   JSON based task format.

dinfitem
>   Double information items

"MSK_DINF_BI_CLEAN_DUAL_TIME"
>   Time spent within the dual clean-up optimizer of the basis identification procedure since its invocation.

"MSK_DINF_BI_CLEAN_PRIMAL_TIME"
>   Time spent within the primal clean-up optimizer of the basis identification procedure since its invocation.

"MSK_DINF_BI_CLEAN_TIME"
>   Time spent within the clean-up phase of the basis identification procedure since its invocation.

"MSK_DINF_BI_DUAL_TIME"
>   Time spent within the dual phase basis identification procedure since its invocation.

"MSK_DINF_BI_PRIMAL_TIME"
>   Time spent within the primal phase of the basis identification procedure since its invocation.

"MSK_DINF_BI_TIME"
>   Time spent within the basis identification procedure since its invocation.

"MSK_DINF_INTPNT_DUAL_FEAS"
>   Dual feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed.)

"MSK_DINF_INTPNT_DUAL_OBJ"
>   Dual objective value reported by the interior-point optimizer.

"MSK_DINF_INTPNT_FACTOR_NUM_FLOPS"
>   An estimate of the number of flops used in the factorization.

"MSK_DINF_INTPNT_OPT_STATUS"
>   A measure of optimality of the solution. It should converge to $+1$ if the problem has a primal-dual optimal solution, and converge to -1 if the problem is (strictly) primal or dual infeasible. If the measure converges to another constant, or fails to settle, the problem is usually ill-posed.

"MSK_DINF_INTPNT_ORDER_TIME"
>   Order time (in seconds).

"MSK_DINF_INTPNT_PRIMAL_FEAS"
>   Primal feasibility measure reported by the interior-point optimizer. (For the interior-point

optimizer this measure is not directly related to the original problem because a homogeneous model is employed).

"MSK_DINF_INTPNT_PRIMAL_OBJ"
Primal objective value reported by the interior-point optimizer.

"MSK_DINF_INTPNT_TIME"
Time spent within the interior-point optimizer since its invocation.

"MSK_DINF_MIO_CLIQUE_SEPARATION_TIME"
Seperation time for clique cuts.

"MSK_DINF_MIO_CMIR_SEPARATION_TIME"
Seperation time for CMIR cuts.

"MSK_DINF_MIO_CONSTRUCT_SOLUTION_OBJ"
If **MOSEK** has successfully constructed an integer feasible solution, then this item contains the optimal objective value corresponding to the feasible solution.

"MSK_DINF_MIO_DUAL_BOUND_AFTER_PRESOLVE"
Value of the dual bound after presolve but before cut generation.

"MSK_DINF_MIO_GMI_SEPARATION_TIME"
Seperation time for GMI cuts.

"MSK_DINF_MIO_HEURISTIC_TIME"
Total time spent in the optimizer.

"MSK_DINF_MIO_IMPLIED_BOUND_TIME"
Seperation time for implied bound cuts.

"MSK_DINF_MIO_KNAPSACK_COVER_SEPARATION_TIME"
Seperation time for knapsack cover.

"MSK_DINF_MIO_OBJ_ABS_GAP"
Given the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the absolute gap defined by

$$|(\text{objective value of feasible solution}) - (\text{objective bound})|.$$

Otherwise it has the value -1.0.

"MSK_DINF_MIO_OBJ_BOUND"
The best known bound on the objective function. This value is undefined until at least one relaxation has been solved: To see if this is the case check that *"MSK_IINF_MIO_NUM_RELAX"* is strictly positive.

"MSK_DINF_MIO_OBJ_INT"
The primal objective value corresponding to the best integer feasible solution. Please note that at least one integer feasible solution must have been located i.e. check *"MSK_IINF_MIO_NUM_INT_SOLUTIONS"*.

"MSK_DINF_MIO_OBJ_REL_GAP"
Given that the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the relative gap defined by

$$\frac{|(\text{objective value of feasible solution}) - (\text{objective bound})|}{\max(\delta, |(\text{objective value of feasible solution})|)}.$$

where δ is given by the parameter *MSK_DPAR_MIO_REL_GAP_CONST*. Otherwise it has the value -1.0.

"MSK_DINF_MIO_OPTIMIZER_TIME"
Total time spent in the optimizer.

"MSK_DINF_MIO_PROBING_TIME"
Total time for probing.

`"MSK_DINF_MIO_ROOT_CUTGEN_TIME"`
  Total time for cut generation.

`"MSK_DINF_MIO_ROOT_OPTIMIZER_TIME"`
  Time spent in the optimizer while solving the root relaxation.

`"MSK_DINF_MIO_ROOT_PRESOLVE_TIME"`
  Time spent in while presolving the root relaxation.

`"MSK_DINF_MIO_TIME"`
  Time spent in the mixed-integer optimizer.

`"MSK_DINF_MIO_USER_OBJ_CUT"`
  If the objective cut is used, then this information item has the value of the cut.

`"MSK_DINF_OPTIMIZER_TIME"`
  Total time spent in the optimizer since it was invoked.

`"MSK_DINF_PRESOLVE_ELI_TIME"`
  Total time spent in the eliminator since the presolve was invoked.

`"MSK_DINF_PRESOLVE_LINDEP_TIME"`
  Total time spent in the linear dependency checker since the presolve was invoked.

`"MSK_DINF_PRESOLVE_TIME"`
  Total time (in seconds) spent in the presolve since it was invoked.

`"MSK_DINF_PRIMAL_REPAIR_PENALTY_OBJ"`
  The optimal objective value of the penalty function.

`"MSK_DINF_QCQO_REFORMULATE_MAX_PERTURBATION"`
  Maximum absolute diagonal perturbation occuring during the QCQO reformulation.

`"MSK_DINF_QCQO_REFORMULATE_TIME"`
  Time spent with conic quadratic reformulation.

`"MSK_DINF_QCQO_REFORMULATE_WORST_CHOLESKY_COLUMN_SCALING"`
  Worst Cholesky column scaling.

`"MSK_DINF_QCQO_REFORMULATE_WORST_CHOLESKY_DIAG_SCALING"`
  Worst Cholesky diagonal scaling.

`"MSK_DINF_RD_TIME"`
  Time spent reading the data file.

`"MSK_DINF_SIM_DUAL_TIME"`
  Time spent in the dual simplex optimizer since invoking it.

`"MSK_DINF_SIM_FEAS"`
  Feasibility measure reported by the simplex optimizer.

`"MSK_DINF_SIM_OBJ"`
  Objective value reported by the simplex optimizer.

`"MSK_DINF_SIM_PRIMAL_TIME"`
  Time spent in the primal simplex optimizer since invoking it.

`"MSK_DINF_SIM_TIME"`
  Time spent in the simplex optimizer since invoking it.

`"MSK_DINF_SOL_BAS_DUAL_OBJ"`
  Dual objective value of the basic solution.

`"MSK_DINF_SOL_BAS_DVIOLCON"`
  Maximal dual bound violation for x^c in the basic solution.

`"MSK_DINF_SOL_BAS_DVIOLVAR"`
  Maximal dual bound violation for x^x in the basic solution.

"MSK_DINF_SOL_BAS_NRM_BARX"
    Infinity norm of $\overline{X}$ in the basic solution.

"MSK_DINF_SOL_BAS_NRM_SLC"
    Infinity norm of s_l^c in the basic solution.

"MSK_DINF_SOL_BAS_NRM_SLX"
    Infinity norm of s_l^x in the basic solution.

"MSK_DINF_SOL_BAS_NRM_SUC"
    Infinity norm of s_u^c in the basic solution.

"MSK_DINF_SOL_BAS_NRM_SUX"
    Infinity norm of s_u^X in the basic solution.

"MSK_DINF_SOL_BAS_NRM_XC"
    Infinity norm of x^c in the basic solution.

"MSK_DINF_SOL_BAS_NRM_XX"
    Infinity norm of x^x in the basic solution.

"MSK_DINF_SOL_BAS_NRM_Y"
    Infinity norm of y in the basic solution.

"MSK_DINF_SOL_BAS_PRIMAL_OBJ"
    Primal objective value of the basic solution.

"MSK_DINF_SOL_BAS_PVIOLCON"
    Maximal primal bound violation for x^c in the basic solution.

"MSK_DINF_SOL_BAS_PVIOLVAR"
    Maximal primal bound violation for x^x in the basic solution.

"MSK_DINF_SOL_ITG_NRM_BARX"
    Infinity norm of $\overline{X}$ in the integer solution.

"MSK_DINF_SOL_ITG_NRM_XC"
    Infinity norm of x^c in the integer solution.

"MSK_DINF_SOL_ITG_NRM_XX"
    Infinity norm of x^x in the integer solution.

"MSK_DINF_SOL_ITG_PRIMAL_OBJ"
    Primal objective value of the integer solution.

"MSK_DINF_SOL_ITG_PVIOLBARVAR"
    Maximal primal bound violation for $\overline{X}$ in the integer solution.

"MSK_DINF_SOL_ITG_PVIOLCON"
    Maximal primal bound violation for x^c in the integer solution.

"MSK_DINF_SOL_ITG_PVIOLCONES"
    Maximal primal violation for primal conic constraints in the integer solution.

"MSK_DINF_SOL_ITG_PVIOLITG"
    Maximal violation for the integer constraints in the integer solution.

"MSK_DINF_SOL_ITG_PVIOLVAR"
    Maximal primal bound violation for x^x in the integer solution.

"MSK_DINF_SOL_ITR_DUAL_OBJ"
    Dual objective value of the interior-point solution.

"MSK_DINF_SOL_ITR_DVIOLBARVAR"
    Maximal dual bound violation for $\overline{X}$ in the interior-point solution.

"MSK_DINF_SOL_ITR_DVIOLCON"
    Maximal dual bound violation for x^c in the interior-point solution.

"MSK_DINF_SOL_ITR_DVIOLCONES"
    Maximal dual violation for dual conic constraints in the interior-point solution.

"MSK_DINF_SOL_ITR_DVIOLVAR"
    Maximal dual bound violation for x^x in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_BARS"
    Infinity norm of $\overline{S}$ in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_BARX"
    Infinity norm of $\overline{X}$ in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_SLC"
    Infinity norm of s_l^c in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_SLX"
    Infinity norm of s_l^x in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_SNX"
    Infinity norm of s_n^x in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_SUC"
    Infinity norm of s_u^c in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_SUX"
    Infinity norm of s_u^X in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_XC"
    Infinity norm of x^c in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_XX"
    Infinity norm of x^x in the interior-point solution.

"MSK_DINF_SOL_ITR_NRM_Y"
    Infinity norm of y in the interior-point solution.

"MSK_DINF_SOL_ITR_PRIMAL_OBJ"
    Primal objective value of the interior-point solution.

"MSK_DINF_SOL_ITR_PVIOLBARVAR"
    Maximal primal bound violation for $\overline{X}$ in the interior-point solution.

"MSK_DINF_SOL_ITR_PVIOLCON"
    Maximal primal bound violation for x^c in the interior-point solution.

"MSK_DINF_SOL_ITR_PVIOLCONES"
    Maximal primal violation for primal conic constraints in the interior-point solution.

"MSK_DINF_SOL_ITR_PVIOLVAR"
    Maximal primal bound violation for x^x in the interior-point solution.

"MSK_DINF_TO_CONIC_TIME"
    Time spent in the last to conic reformulation.

feature
    License feature

"MSK_FEATURE_PTS"
    Base system.

"MSK_FEATURE_PTON"
    Nonlinear extension.

liinfitem
    Long integer information items.

"MSK_LIINF_BI_CLEAN_DUAL_DEG_ITER"
    Number of dual degenerate clean iterations performed in the basis identification.

"MSK_LIINF_BI_CLEAN_DUAL_ITER"
    Number of dual clean iterations performed in the basis identification.

"MSK_LIINF_BI_CLEAN_PRIMAL_DEG_ITER"
    Number of primal degenerate clean iterations performed in the basis identification.

"MSK_LIINF_BI_CLEAN_PRIMAL_ITER"
    Number of primal clean iterations performed in the basis identification.

"MSK_LIINF_BI_DUAL_ITER"
    Number of dual pivots performed in the basis identification.

"MSK_LIINF_BI_PRIMAL_ITER"
    Number of primal pivots performed in the basis identification.

"MSK_LIINF_INTPNT_FACTOR_NUM_NZ"
    Number of non-zeros in factorization.

"MSK_LIINF_MIO_INTPNT_ITER"
    Number of interior-point iterations performed by the mixed-integer optimizer.

"MSK_LIINF_MIO_PRESOLVED_ANZ"
    Number of non-zero entries in the constraint matrix of presolved problem.

"MSK_LIINF_MIO_SIM_MAXITER_SETBACKS"
    Number of times the the simplex optimizer has hit the maximum iteration limit when re-optimizing.

"MSK_LIINF_MIO_SIMPLEX_ITER"
    Number of simplex iterations performed by the mixed-integer optimizer.

"MSK_LIINF_RD_NUMANZ"
    Number of non-zeros in A that is read.

"MSK_LIINF_RD_NUMQNZ"
    Number of Q non-zeros.

iinfitem
    Integer information items.

"MSK_IINF_ANA_PRO_NUM_CON"
    Number of constraints in the problem.

"MSK_IINF_ANA_PRO_NUM_CON_EQ"
    Number of equality constraints.

"MSK_IINF_ANA_PRO_NUM_CON_FR"
    Number of unbounded constraints.

"MSK_IINF_ANA_PRO_NUM_CON_LO"
    Number of constraints with a lower bound and an infinite upper bound.

"MSK_IINF_ANA_PRO_NUM_CON_RA"
    Number of constraints with finite lower and upper bounds.

"MSK_IINF_ANA_PRO_NUM_CON_UP"
    Number of constraints with an upper bound and an infinite lower bound.

"MSK_IINF_ANA_PRO_NUM_VAR"
    Number of variables in the problem.

"MSK_IINF_ANA_PRO_NUM_VAR_BIN"
    Number of binary (0-1) variables.

"MSK_IINF_ANA_PRO_NUM_VAR_CONT"
    Number of continuous variables.

"MSK_IINF_ANA_PRO_NUM_VAR_EQ"
    Number of fixed variables.

`"MSK_IINF_ANA_PRO_NUM_VAR_FR"`
    Number of free variables.

`"MSK_IINF_ANA_PRO_NUM_VAR_INT"`
    Number of general integer variables.

`"MSK_IINF_ANA_PRO_NUM_VAR_LO"`
    Number of variables with a lower bound and an infinite upper bound.

`"MSK_IINF_ANA_PRO_NUM_VAR_RA"`
    Number of variables with finite lower and upper bounds.

`"MSK_IINF_ANA_PRO_NUM_VAR_UP"`
    Number of variables with an upper bound and an infinite lower bound. This value is set by

`"MSK_IINF_INTPNT_FACTOR_DIM_DENSE"`
    Dimension of the dense sub system in factorization.

`"MSK_IINF_INTPNT_ITER"`
    Number of interior-point iterations since invoking the interior-point optimizer.

`"MSK_IINF_INTPNT_NUM_THREADS"`
    Number of threads that the interior-point optimizer is using.

`"MSK_IINF_INTPNT_SOLVE_DUAL"`
    Non-zero if the interior-point optimizer is solving the dual problem.

`"MSK_IINF_MIO_ABSGAP_SATISFIED"`
    Non-zero if absolute gap is within tolerances.

`"MSK_IINF_MIO_CLIQUE_TABLE_SIZE"`
    Size of the clique table.

`"MSK_IINF_MIO_CONSTRUCT_NUM_ROUNDINGS"`
    Number of values in the integer solution that is rounded to an integer value.

`"MSK_IINF_MIO_CONSTRUCT_SOLUTION"`
    If this item has the value 0, then **MOSEK** did not try to construct an initial integer feasible solution. If the item has a positive value, then **MOSEK** successfully constructed an initial integer feasible solution.

`"MSK_IINF_MIO_INITIAL_SOLUTION"`
    Is non-zero if an initial integer solution is specified.

`"MSK_IINF_MIO_NEAR_ABSGAP_SATISFIED"`
    Non-zero if absolute gap is within relaxed tolerances.

`"MSK_IINF_MIO_NEAR_RELGAP_SATISFIED"`
    Non-zero if relative gap is within relaxed tolerances.

`"MSK_IINF_MIO_NODE_DEPTH"`
    Depth of the last node solved.

`"MSK_IINF_MIO_NUM_ACTIVE_NODES"`
    Number of active branch bound nodes.

`"MSK_IINF_MIO_NUM_BRANCH"`
    Number of branches performed during the optimization.

`"MSK_IINF_MIO_NUM_CLIQUE_CUTS"`
    Number of clique cuts.

`"MSK_IINF_MIO_NUM_CMIR_CUTS"`
    Number of Complemented Mixed Integer Rounding (CMIR) cuts.

`"MSK_IINF_MIO_NUM_GOMORY_CUTS"`
    Number of Gomory cuts.

"MSK_IINF_MIO_NUM_IMPLIED_BOUND_CUTS"
    Number of implied bound cuts.

"MSK_IINF_MIO_NUM_INT_SOLUTIONS"
    Number of integer feasible solutions that has been found.

"MSK_IINF_MIO_NUM_KNAPSACK_COVER_CUTS"
    Number of clique cuts.

"MSK_IINF_MIO_NUM_RELAX"
    Number of relaxations solved during the optimization.

"MSK_IINF_MIO_NUM_REPEATED_PRESOLVE"
    Number of times presolve was repeated at root.

"MSK_IINF_MIO_NUMCON"
    Number of constraints in the problem solved by the mixed-integer optimizer.

"MSK_IINF_MIO_NUMINT"
    Number of integer variables in the problem solved be the mixed-integer optimizer.

"MSK_IINF_MIO_NUMVAR"
    Number of variables in the problem solved by the mixed-integer optimizer.

"MSK_IINF_MIO_OBJ_BOUND_DEFINED"
    Non-zero if a valid objective bound has been found, otherwise zero.

"MSK_IINF_MIO_PRESOLVED_NUMBIN"
    Number of binary variables in the problem solved be the mixed-integer optimizer.

"MSK_IINF_MIO_PRESOLVED_NUMCON"
    Number of constraints in the presolved problem.

"MSK_IINF_MIO_PRESOLVED_NUMCONT"
    Number of continuous variables in the problem solved be the mixed-integer optimizer.

"MSK_IINF_MIO_PRESOLVED_NUMINT"
    Number of integer variables in the presolved problem.

"MSK_IINF_MIO_PRESOLVED_NUMVAR"
    Number of variables in the presolved problem.

"MSK_IINF_MIO_RELGAP_SATISFIED"
    Non-zero if relative gap is within tolerances.

"MSK_IINF_MIO_TOTAL_NUM_CUTS"
    Total number of cuts generated by the mixed-integer optimizer.

"MSK_IINF_MIO_USER_OBJ_CUT"
    If it is non-zero, then the objective cut is used.

"MSK_IINF_OPT_NUMCON"
    Number of constraints in the problem solved when the optimizer is called.

"MSK_IINF_OPT_NUMVAR"
    Number of variables in the problem solved when the optimizer is called

"MSK_IINF_OPTIMIZE_RESPONSE"
    The response code returned by optimize.

"MSK_IINF_RD_NUMBARVAR"
    Number of variables read.

"MSK_IINF_RD_NUMCON"
    Number of constraints read.

"MSK_IINF_RD_NUMCONE"
    Number of conic constraints read.

`"MSK_IINF_RD_NUMINTVAR"`
> Number of integer-constrained variables read.

`"MSK_IINF_RD_NUMQ"`
> Number of nonempty Q matrices read.

`"MSK_IINF_RD_NUMVAR"`
> Number of variables read.

`"MSK_IINF_RD_PROTYPE"`
> Problem type.

`"MSK_IINF_SIM_DUAL_DEG_ITER"`
> The number of dual degenerate iterations.

`"MSK_IINF_SIM_DUAL_HOTSTART"`
> If 1 then the dual simplex algorithm is solving from an advanced basis.

`"MSK_IINF_SIM_DUAL_HOTSTART_LU"`
> If 1 then a valid basis factorization of full rank was located and used by the dual simplex algorithm.

`"MSK_IINF_SIM_DUAL_INF_ITER"`
> The number of iterations taken with dual infeasibility.

`"MSK_IINF_SIM_DUAL_ITER"`
> Number of dual simplex iterations during the last optimization.

`"MSK_IINF_SIM_NUMCON"`
> Number of constraints in the problem solved by the simplex optimizer.

`"MSK_IINF_SIM_NUMVAR"`
> Number of variables in the problem solved by the simplex optimizer.

`"MSK_IINF_SIM_PRIMAL_DEG_ITER"`
> The number of primal degenerate iterations.

`"MSK_IINF_SIM_PRIMAL_HOTSTART"`
> If 1 then the primal simplex algorithm is solving from an advanced basis.

`"MSK_IINF_SIM_PRIMAL_HOTSTART_LU"`
> If 1 then a valid basis factorization of full rank was located and used by the primal simplex algorithm.

`"MSK_IINF_SIM_PRIMAL_INF_ITER"`
> The number of iterations taken with primal infeasibility.

`"MSK_IINF_SIM_PRIMAL_ITER"`
> Number of primal simplex iterations during the last optimization.

`"MSK_IINF_SIM_SOLVE_DUAL"`
> Is non-zero if dual problem is solved.

`"MSK_IINF_SOL_BAS_PROSTA"`
> Problem status of the basic solution. Updated after each optimization.

`"MSK_IINF_SOL_BAS_SOLSTA"`
> Solution status of the basic solution. Updated after each optimization.

`"MSK_IINF_SOL_ITG_PROSTA"`
> Problem status of the integer solution. Updated after each optimization.

`"MSK_IINF_SOL_ITG_SOLSTA"`
> Solution status of the integer solution. Updated after each optimization.

`"MSK_IINF_SOL_ITR_PROSTA"`
> Problem status of the interior-point solution. Updated after each optimization.

"MSK_IINF_SOL_ITR_SOLSTA"
    Solution status of the interior-point solution. Updated after each optimization.

"MSK_IINF_STO_NUM_A_REALLOC"
    Number of times the storage for storing A has been changed. A large value may indicates that memory fragmentation may occur.

inftype
    Information item types

"MSK_INF_DOU_TYPE"
    Is a double information type.

"MSK_INF_INT_TYPE"
    Is an integer.

"MSK_INF_LINT_TYPE"
    Is a long integer.

iomode
    Input/output modes

"MSK_IOMODE_READ"
    The file is read-only.

"MSK_IOMODE_WRITE"
    The file is write-only. If the file exists then it is truncated when it is opened. Otherwise it is created when it is opened.

"MSK_IOMODE_READWRITE"
    The file is to read and written.

branchdir
    Specifies the branching direction.

"MSK_BRANCH_DIR_FREE"
    The mixed-integer optimizer decides which branch to choose.

"MSK_BRANCH_DIR_UP"
    The mixed-integer optimizer always chooses the up branch first.

"MSK_BRANCH_DIR_DOWN"
    The mixed-integer optimizer always chooses the down branch first.

"MSK_BRANCH_DIR_NEAR"
    Branch in direction nearest to selected fractional variable.

"MSK_BRANCH_DIR_FAR"
    Branch in direction farthest from selected fractional variable.

"MSK_BRANCH_DIR_ROOT_LP"
    Chose direction based on root lp value of selected variable.

"MSK_BRANCH_DIR_GUIDED"
    Branch in direction of current incumbent.

"MSK_BRANCH_DIR_PSEUDOCOST"
    Branch based on the pseudocost of the variable.

miocontsoltype
    Continuous mixed-integer solution type

"MSK_MIO_CONT_SOL_NONE"
    No interior-point or basic solution are reported when the mixed-integer optimizer is used.

"MSK_MIO_CONT_SOL_ROOT"
    The reported interior-point and basic solutions are a solution to the root node problem when mixed-integer optimizer is used.

"MSK_MIO_CONT_SOL_ITG"
> The reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. A solution is only reported in case the problem has a primal feasible solution.

"MSK_MIO_CONT_SOL_ITG_REL"
> In case the problem is primal feasible then the reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. If the problem is primal infeasible, then the solution to the root node problem is reported.

miomode
> Integer restrictions

"MSK_MIO_MODE_IGNORED"
> The integer constraints are ignored and the problem is solved as a continuous problem.

"MSK_MIO_MODE_SATISFIED"
> Integer restrictions should be satisfied.

mionodeseltype
> Mixed-integer node selection types

"MSK_MIO_NODE_SELECTION_FREE"
> The optimizer decides the node selection strategy.

"MSK_MIO_NODE_SELECTION_FIRST"
> The optimizer employs a depth first node selection strategy.

"MSK_MIO_NODE_SELECTION_BEST"
> The optimizer employs a best bound node selection strategy.

"MSK_MIO_NODE_SELECTION_WORST"
> The optimizer employs a worst bound node selection strategy.

"MSK_MIO_NODE_SELECTION_HYBRID"
> The optimizer employs a hybrid strategy.

"MSK_MIO_NODE_SELECTION_PSEUDO"
> The optimizer employs selects the node based on a pseudo cost estimate.

mpsformat
> MPS file format type

"MSK_MPS_FORMAT_STRICT"
> It is assumed that the input file satisfies the MPS format strictly.

"MSK_MPS_FORMAT_RELAXED"
> It is assumed that the input file satisfies a slightly relaxed version of the MPS format.

"MSK_MPS_FORMAT_FREE"
> It is assumed that the input file satisfies the free MPS format. This implies that spaces are not allowed in names. Otherwise the format is free.

"MSK_MPS_FORMAT_CPLEX"
> The CPLEX compatible version of the MPS format is employed.

objsense
> Objective sense types

"MSK_OBJECTIVE_SENSE_MINIMIZE"
> The problem should be minimized.

"MSK_OBJECTIVE_SENSE_MAXIMIZE"
> The problem should be maximized.

onoffkey
> On/off

"MSK_ON"
> Switch the option on.

"MSK_OFF"
> Switch the option off.

optimizertype
> Optimizer types

"MSK_OPTIMIZER_CONIC"
> The optimizer for problems having conic constraints.

"MSK_OPTIMIZER_DUAL_SIMPLEX"
> The dual simplex optimizer is used.

"MSK_OPTIMIZER_FREE"
> The optimizer is chosen automatically.

"MSK_OPTIMIZER_FREE_SIMPLEX"
> One of the simplex optimizers is used.

"MSK_OPTIMIZER_INTPNT"
> The interior-point optimizer is used.

"MSK_OPTIMIZER_MIXED_INT"
> The mixed-integer optimizer.

"MSK_OPTIMIZER_PRIMAL_SIMPLEX"
> The primal simplex optimizer is used.

orderingtype
> Ordering strategies

"MSK_ORDER_METHOD_FREE"
> The ordering method is chosen automatically.

"MSK_ORDER_METHOD_APPMINLOC"
> Approximate minimum local fill-in ordering is employed.

"MSK_ORDER_METHOD_EXPERIMENTAL"
> This option should not be used.

"MSK_ORDER_METHOD_TRY_GRAPHPAR"
> Always try the graph partitioning based ordering.

"MSK_ORDER_METHOD_FORCE_GRAPHPAR"
> Always use the graph partitioning based ordering even if it is worse than the approximate minimum local fill ordering.

"MSK_ORDER_METHOD_NONE"
> No ordering is used.

presolvemode
> Presolve method.

"MSK_PRESOLVE_MODE_OFF"
> The problem is not presolved before it is optimized.

"MSK_PRESOLVE_MODE_ON"
> The problem is presolved before it is optimized.

"MSK_PRESOLVE_MODE_FREE"
> It is decided automatically whether to presolve before the problem is optimized.

parametertype
> Parameter type

"MSK_PAR_INVALID_TYPE"
> Not a valid parameter.

"MSK_PAR_DOU_TYPE"
  Is a double parameter.

"MSK_PAR_INT_TYPE"
  Is an integer parameter.

"MSK_PAR_STR_TYPE"
  Is a string parameter.

problemitem
  Problem data items

"MSK_PI_VAR"
  Item is a variable.

"MSK_PI_CON"
  Item is a constraint.

"MSK_PI_CONE"
  Item is a cone.

problemtype
  Problem types

"MSK_PROBTYPE_LO"
  The problem is a linear optimization problem.

"MSK_PROBTYPE_QO"
  The problem is a quadratic optimization problem.

"MSK_PROBTYPE_QCQO"
  The problem is a quadratically constrained optimization problem.

"MSK_PROBTYPE_GECO"
  General convex optimization.

"MSK_PROBTYPE_CONIC"
  A conic optimization.

"MSK_PROBTYPE_MIXED"
  General nonlinear constraints and conic constraints. This combination can not be solved by **MOSEK**.

prosta
  Problem status keys

"MSK_PRO_STA_UNKNOWN"
  Unknown problem status.

"MSK_PRO_STA_PRIM_AND_DUAL_FEAS"
  The problem is primal and dual feasible.

"MSK_PRO_STA_PRIM_FEAS"
  The problem is primal feasible.

"MSK_PRO_STA_DUAL_FEAS"
  The problem is dual feasible.

"MSK_PRO_STA_NEAR_PRIM_AND_DUAL_FEAS"
  The problem is at least nearly primal and dual feasible.

"MSK_PRO_STA_NEAR_PRIM_FEAS"
  The problem is at least nearly primal feasible.

"MSK_PRO_STA_NEAR_DUAL_FEAS"
  The problem is at least nearly dual feasible.

"MSK_PRO_STA_PRIM_INFEAS"
  The problem is primal infeasible.

12.5. Enumerations

"MSK_PRO_STA_DUAL_INFEAS"
> The problem is dual infeasible.

"MSK_PRO_STA_PRIM_AND_DUAL_INFEAS"
> The problem is primal and dual infeasible.

"MSK_PRO_STA_ILL_POSED"
> The problem is ill-posed. For example, it may be primal and dual feasible but have a positive duality gap.

"MSK_PRO_STA_PRIM_INFEAS_OR_UNBOUNDED"
> The problem is either primal infeasible or unbounded. This may occur for mixed-integer problems.

xmlwriteroutputtype
> XML writer output mode

"MSK_WRITE_XML_MODE_ROW"
> Write in row order.

"MSK_WRITE_XML_MODE_COL"
> Write in column order.

rescodetype
> Response code type

"MSK_RESPONSE_OK"
> The response code is OK.

"MSK_RESPONSE_WRN"
> The response code is a warning.

"MSK_RESPONSE_TRM"
> The response code is an optimizer termination status.

"MSK_RESPONSE_ERR"
> The response code is an error.

"MSK_RESPONSE_UNK"
> The response code does not belong to any class.

scalingtype
> Scaling type

"MSK_SCALING_FREE"
> The optimizer chooses the scaling heuristic.

"MSK_SCALING_NONE"
> No scaling is performed.

"MSK_SCALING_MODERATE"
> A conservative scaling is performed.

"MSK_SCALING_AGGRESSIVE"
> A very aggressive scaling is performed.

scalingmethod
> Scaling method

"MSK_SCALING_METHOD_POW2"
> Scales only with power of 2 leaving the mantissa untouched.

"MSK_SCALING_METHOD_FREE"
> The optimizer chooses the scaling heuristic.

sensitivitytype
> Sensitivity types

"MSK_SENSITIVITY_TYPE_BASIS"
> Basis sensitivity analysis is performed.

"MSK_SENSITIVITY_TYPE_OPTIMAL_PARTITION"
> Optimal partition sensitivity analysis is performed.

simseltype
> Simplex selection strategy

"MSK_SIM_SELECTION_FREE"
> The optimizer chooses the pricing strategy.

"MSK_SIM_SELECTION_FULL"
> The optimizer uses full pricing.

"MSK_SIM_SELECTION_ASE"
> The optimizer uses approximate steepest-edge pricing.

"MSK_SIM_SELECTION_DEVEX"
> The optimizer uses devex steepest-edge pricing (or if it is not available an approximate steep-edge selection).

"MSK_SIM_SELECTION_SE"
> The optimizer uses steepest-edge selection (or if it is not available an approximate steep-edge selection).

"MSK_SIM_SELECTION_PARTIAL"
> The optimizer uses a partial selection approach. The approach is usually beneficial if the number of variables is much larger than the number of constraints.

solitem
> Solution items

"MSK_SOL_ITEM_XC"
> Solution for the constraints.

"MSK_SOL_ITEM_XX"
> Variable solution.

"MSK_SOL_ITEM_Y"
> Lagrange multipliers for equations.

"MSK_SOL_ITEM_SLC"
> Lagrange multipliers for lower bounds on the constraints.

"MSK_SOL_ITEM_SUC"
> Lagrange multipliers for upper bounds on the constraints.

"MSK_SOL_ITEM_SLX"
> Lagrange multipliers for lower bounds on the variables.

"MSK_SOL_ITEM_SUX"
> Lagrange multipliers for upper bounds on the variables.

"MSK_SOL_ITEM_SNX"
> Lagrange multipliers corresponding to the conic constraints on the variables.

solsta
> Solution status keys

"MSK_SOL_STA_UNKNOWN"
> Status of the solution is unknown.

"MSK_SOL_STA_OPTIMAL"
> The solution is optimal.

"MSK_SOL_STA_PRIM_FEAS"
> The solution is primal feasible.

    `"MSK_SOL_STA_DUAL_FEAS"`
        The solution is dual feasible.

    `"MSK_SOL_STA_PRIM_AND_DUAL_FEAS"`
        The solution is both primal and dual feasible.

    `"MSK_SOL_STA_NEAR_OPTIMAL"`
        The solution is nearly optimal.

    `"MSK_SOL_STA_NEAR_PRIM_FEAS"`
        The solution is nearly primal feasible.

    `"MSK_SOL_STA_NEAR_DUAL_FEAS"`
        The solution is nearly dual feasible.

    `"MSK_SOL_STA_NEAR_PRIM_AND_DUAL_FEAS"`
        The solution is nearly both primal and dual feasible.

    `"MSK_SOL_STA_PRIM_INFEAS_CER"`
        The solution is a certificate of primal infeasibility.

    `"MSK_SOL_STA_DUAL_INFEAS_CER"`
        The solution is a certificate of dual infeasibility.

    `"MSK_SOL_STA_NEAR_PRIM_INFEAS_CER"`
        The solution is almost a certificate of primal infeasibility.

    `"MSK_SOL_STA_NEAR_DUAL_INFEAS_CER"`
        The solution is almost a certificate of dual infeasibility.

    `"MSK_SOL_STA_PRIM_ILLPOSED_CER"`
        The solution is a certificate that the primal problem is illposed.

    `"MSK_SOL_STA_DUAL_ILLPOSED_CER"`
        The solution is a certificate that the dual problem is illposed.

    `"MSK_SOL_STA_INTEGER_OPTIMAL"`
        The primal solution is integer optimal.

    `"MSK_SOL_STA_NEAR_INTEGER_OPTIMAL"`
        The primal solution is near integer optimal.

`soltype`
    Solution types

    `"MSK_SOL_BAS"`
        The basic solution.

    `"MSK_SOL_ITR"`
        The interior solution.

    `"MSK_SOL_ITG"`
        The integer solution.

`solveform`
    Solve primal or dual form

    `"MSK_SOLVE_FREE"`
        The optimizer is free to solve either the primal or the dual problem.

    `"MSK_SOLVE_PRIMAL"`
        The optimizer should solve the primal problem.

    `"MSK_SOLVE_DUAL"`
        The optimizer should solve the dual problem.

`stakey`
    Status keys

`"MSK_SK_UNK"`
>   The status for the constraint or variable is unknown.

`"MSK_SK_BAS"`
>   The constraint or variable is in the basis.

`"MSK_SK_SUPBAS"`
>   The constraint or variable is super basic.

`"MSK_SK_LOW"`
>   The constraint or variable is at its lower bound.

`"MSK_SK_UPR"`
>   The constraint or variable is at its upper bound.

`"MSK_SK_FIX"`
>   The constraint or variable is fixed.

`"MSK_SK_INF"`
>   The constraint or variable is infeasible in the bounds.

`startpointtype`
>   Starting point types

`"MSK_STARTING_POINT_FREE"`
>   The starting point is chosen automatically.

`"MSK_STARTING_POINT_GUESS"`
>   The optimizer guesses a starting point.

`"MSK_STARTING_POINT_CONSTANT"`
>   The optimizer constructs a starting point by assigning a constant value to all primal and dual variables. This starting point is normally robust.

`"MSK_STARTING_POINT_SATISFY_BOUNDS"`
>   The starting point is chosen to satisfy all the simple bounds on nonlinear variables. If this starting point is employed, then more care than usual should employed when choosing the bounds on the nonlinear variables. In particular very tight bounds should be avoided.

`streamtype`
>   Stream types

`"MSK_STREAM_LOG"`
>   Log stream. Contains the aggregated contents of all other streams. This means that a message written to any other stream will also be written to this stream.

`"MSK_STREAM_MSG"`
>   Message stream. Log information relating to performance and progress of the optimization is written to this stream.

`"MSK_STREAM_ERR"`
>   Error stream. Error messages are written to this stream.

`"MSK_STREAM_WRN"`
>   Warning stream. Warning messages are written to this stream.

`value`
>   Integer values

`"MSK_MAX_STR_LEN"`
>   Maximum string length allowed in **MOSEK**.

`"MSK_LICENSE_BUFFER_LENGTH"`
>   The length of a license key buffer.

`variabletype`
>   Variable types

`"MSK_VAR_TYPE_CONT"`
Is a continuous variable.

`"MSK_VAR_TYPE_INT"`
Is an integer variable.

SUPPORTED FILE FORMATS

MOSEK supports a range of problem and solution formats listed in Table 13.1 and Table 13.2. The **Task format** is **MOSEK**'s native binary format and it supports all features that **MOSEK** supports. The **OPF format** is **MOSEK**'s human-readable alternative that supports nearly all features (everything except semidefinite problems). In general, text formats are significantly slower to read, but can be examined and edited directly in any text editor.

Problem formats

See Table 13.1.

Table 13.1: List of supported file formats for optimization problems.

Format Type	Ext.	Binary/Text	LP	QO	CQO	SDP
LP	lp	plain text	X	X		
MPS	mps	plain text	X	X		
OPF	opf	plain text	X	X	X	
CBF	cbf	plain text	X		X	X
OSiL	xml	xml text	X	X		
Task format	task	binary	X	X	X	X
Jtask format	jtask	text	X	X	X	X

Solution formats

See Table 13.2.

Table 13.2: List of supported solution formats.

Format Type	Ext.	Binary/Text	Description
SOL	sol	plain text	Interior Solution
	bas	plain text	Basic Solution
	int	plain text	Integer
Jsol format	jsol	text	Solution

Compression

MOSEK supports GZIP compression of files. Problem files with an additional `.gz` extension are assumed to be compressed when read, and are automatically compressed when written. For example, a file called

```
problem.mps.gz
```

will be considered as a GZIP compressed MPS file.

13.1 The LP File Format

MOSEK supports the LP file format with some extensions. The LP format is not a completely well-defined standard and hence different optimization packages may interpret the same LP file in slightly different ways. **MOSEK** tries to emulate as closely as possible CPLEX's behavior, but tries to stay backward compatible.

The LP file format can specify problems on the form

$$
\begin{array}{lrcl}
\text{minimize/maximize} & & c^T x + \frac{1}{2} q^o(x) & \\
\text{subject to} & l^c \leq & Ax + \frac{1}{2} q(x) & \leq u^c, \\
& l^x \leq & x & \leq u^x, \\
& & x_{\mathcal{J}} \text{ integer}, &
\end{array}
$$

where

- $x \in \mathbb{R}^n$ is the vector of decision variables.

- $c \in \mathbb{R}^n$ is the linear term in the objective.

- $q^o :\in \mathbb{R}^n \to \mathbb{R}$ is the quadratic term in the objective where

$$q^o(x) = x^T Q^o x$$

and it is assumed that

$$Q^o = (Q^o)^T.$$

- $A \in \mathbb{R}^{m \times n}$ is the constraint matrix.

- $l^c \in \mathbb{R}^m$ is the lower limit on the activity for the constraints.

- $u^c \in \mathbb{R}^m$ is the upper limit on the activity for the constraints.

- $l^x \in \mathbb{R}^n$ is the lower limit on the activity for the variables.

- $u^x \in \mathbb{R}^n$ is the upper limit on the activity for the variables.

- $q : \mathbb{R}^n \to \mathbb{R}$ is a vector of quadratic functions. Hence,

$$q_i(x) = x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T.$$

- $\mathcal{J} \subseteq \{1, 2, \ldots, n\}$ is an index set of the integer constrained variables.

13.1.1 File Sections

An LP formatted file contains a number of sections specifying the objective, constraints, variable bounds, and variable types. The section keywords may be any mix of upper and lower case letters.

Objective Function

The first section beginning with one of the keywords

```
max
maximum
maximize
min
minimum
minimize
```

defines the objective sense and the objective function, i.e.

$$c^T x + \frac{1}{2} x^T Q^o x.$$

The objective may be given a name by writing

```
myname:
```

before the expressions. If no name is given, then the objective is named `obj`.

The objective function contains linear and quadratic terms. The linear terms are written as:

```
4 x1 + x2 - 0.1 x3
```

and so forth. The quadratic terms are written in square brackets (`[ ]`) and are either squared or multiplied as in the examples

```
x1^2
```

and

```
x1 * x2
```

There may be zero or more pairs of brackets containing quadratic expressions.

An example of an objective section is

```
minimize
myobj: 4 x1 + x2 - 0.1 x3 + [ x1^2 + 2.1 x1 * x2 ]/2
```

Please note that the quadratic expressions are multiplied with $\frac{1}{2}$, so that the above expression means

$$\text{minimize} \quad 4x_1 + x_2 - 0.1 \cdot x_3 + \tfrac{1}{2}(x_1^2 + 2.1 \cdot x_1 \cdot x_2)$$

If the same variable occurs more than once in the linear part, the coefficients are added, so that `4 x1 + 2 x1` is equivalent to `6 x1`. In the quadratic expressions `x1 * x2` is equivalent to `x2 * x1` and, as in the linear part, if the same variables multiplied or squared occur several times their coefficients are added.

Constraints

The second section beginning with one of the keywords

```
subj to
subject to
s.t.
st
```

defines the linear constraint matrix A and the quadratic matrices Q^i.

A constraint contains a name (optional), expressions adhering to the same rules as in the objective and a bound:

```
subject to
con1: x1 + x2 + [ x3^2 ]/2 <= 5.1
```

The bound type (here <=) may be any of <, <=, =, >, >= (< and <= mean the same), and the bound may be any number.

In the standard LP format it is not possible to define more than one bound, but **MOSEK** supports defining ranged constraints by using double-colon (::) instead of a single-colon (:) after the constraint name, i.e.

$$-5 \leq x_1 + x_2 \leq 5 \tag{13.1}$$

may be written as

```
con:: -5 < x_1 + x_2 < 5
```

By default **MOSEK** writes ranged constraints this way.

If the files must adhere to the LP standard, ranged constraints must either be split into upper bounded and lower bounded constraints or be written as an equality with a slack variable. For example the expression (13.1) may be written as

$$x_1 + x_2 - sl_1 = 0, \ -5 \leq sl_1 \leq 5.$$

Bounds

Bounds on the variables can be specified in the bound section beginning with one of the keywords

```
bound
bounds
```

The bounds section is optional but should, if present, follow the `subject to` section. All variables listed in the bounds section must occur in either the objective or a constraint.

The default lower and upper bounds are 0 and $+\infty$. A variable may be declared free with the keyword `free`, which means that the lower bound is $-\infty$ and the upper bound is $+\infty$. Furthermore it may be assigned a finite lower and upper bound. The bound definitions for a given variable may be written in one or two lines, and bounds can be any number or $\pm\infty$ (written as `+inf`/`-inf`/`+infinity`/`-infinity`) as in the example

```
bounds
x1 free
x2 <= 5
0.1 <= x2
x3 = 42
2 <= x4 < +inf
```

Variable Types

The final two sections are optional and must begin with one of the keywords

```
bin
binaries
binary
```

and

```
gen
general
```

Under `general` all integer variables are listed, and under `binary` all binary (integer variables with bounds 0 and 1) are listed:

```
general
x1 x2
binary
x3 x4
```

Again, all variables listed in the binary or general sections must occur in either the objective or a constraint.

Terminating Section

Finally, an LP formatted file must be terminated with the keyword

```
end
```

13.1.2 LP File Examples

Linear example `lo1.lp`

```
\ File: lo1.lp
maximize
obj: 3 x1 + x2 + 5 x3 + x4
subject to
c1:  3 x1 + x2 + 2 x3 = 30
c2:  2 x1 + x2 + 3 x3 + x4 >= 15
c3:  2 x2 + 3 x4 <= 25
bounds
 0 <= x1 <= +infinity
 0 <= x2 <= 10
 0 <= x3 <= +infinity
 0 <= x4 <= +infinity
end
```

Mixed integer example `milo1.lp`

```
maximize
obj: x1 + 6.4e-01 x2
subject to
c1:  5e+01 x1 + 3.1e+01 x2 <= 2.5e+02
c2:  3e+00 x1 - 2e+00 x2 >= -4e+00
bounds
 0 <= x1 <= +infinity
 0 <= x2 <= +infinity
general
 x1 x2
end
```

13.1.3 LP Format peculiarities

Comments

Anything on a line after a \ is ignored and is treated as a comment.

Names

A name for an objective, a constraint or a variable may contain the letters *a-z*, *A-Z*, the digits *0-9* and the characters

```
!"#$%&()/,.;?@_'`|~
```

The first character in a name must not be a number, a period or the letter `e` or `E`. Keywords must not be used as names.

MOSEK accepts any character as valid for names, except `\0`. A name that is not allowed in LP file will be changed and a warning will be issued.

The algorithm for making names LP valid works as follows: The name is interpreted as an `utf-8` string. For a unicode character `c`:

- If `c==_` (underscore), the output is `__` (two underscores).

- If `c` is a valid LP name character, the output is just `c`.

- If `c` is another character in the ASCII range, the output is `_XX`, where `XX` is the hexadecimal code for the character.

- If `c` is a character in the range *127-65535*, the output is `_uXXXX`, where `XXXX` is the hexadecimal code for the character.

- If `c` is a character above 65535, the output is `_UXXXXXXXX`, where `XXXXXXXX` is the hexadecimal code for the character.

Invalid `utf-8` substrings are escaped as `_XX'`, and if a name starts with a period, *e* or *E*, that character is escaped as `_XX`.

Variable Bounds

Specifying several upper or lower bounds on one variable is possible but **MOSEK** uses only the tightest bounds. If a variable is fixed (with `=`), then it is considered the tightest bound.

MOSEK Extensions to the LP Format

Some optimization software packages employ a more strict definition of the LP format than the one used by **MOSEK**. The limitations imposed by the strict LP format are the following:

- Quadratic terms in the constraints are not allowed.

- Names can be only 16 characters long.

- Lines must not exceed 255 characters in length.

If an LP formatted file created by **MOSEK** should satisfy the strict definition, then the parameter

- *MSK_IPAR_WRITE_LP_STRICT_FORMAT*

should be set; note, however, that some problems cannot be written correctly as a strict LP formatted file. For instance, all names are truncated to 16 characters and hence they may loose their uniqueness and change the problem.

To get around some of the inconveniences converting from other problem formats, **MOSEK** allows lines to contain 1024 characters and names may have any length (shorter than the 1024 characters).

Internally in **MOSEK** names may contain any (printable) character, many of which cannot be used in LP names. Setting the parameters

- *MSK_IPAR_READ_LP_QUOTED_NAMES* and

- *MSK_IPAR_WRITE_LP_QUOTED_NAMES*

allows **MOSEK** to use quoted names. The first parameter tells **MOSEK** to remove quotes from quoted names e.g, `"x1"`, when reading LP formatted files. The second parameter tells **MOSEK** to put quotes around any semi-illegal name (names beginning with a number or a period) and fully illegal name (containing illegal characters). As double quote is a legal character in the LP format, quoting semi-illegal names makes them legal in the pure LP format as long as they are still shorter than 16 characters. Fully illegal names are still illegal in a pure LP file.

13.1.4 The strict LP format

The LP format is not a formal standard and different vendors have slightly different interpretations of the LP format. To make **MOSEK**'s definition of the LP format more compatible with the definitions of other vendors, use the parameter setting

- *MSK_IPAR_WRITE_LP_STRICT_FORMAT = "MSK_ON"*

This setting may lead to truncation of some names and hence to an invalid LP file. The simple solution to this problem is to use the parameter setting

- *MSK_IPAR_WRITE_GENERIC_NAMES = "MSK_ON"*

which will cause all names to be renamed systematically in the output file.

13.1.5 Formatting of an LP File

A few parameters control the visual formatting of LP files written by **MOSEK** in order to make it easier to read the files. These parameters are

- *MSK_IPAR_WRITE_LP_LINE_WIDTH*

- *MSK_IPAR_WRITE_LP_TERMS_PER_LINE*

The first parameter sets the maximum number of characters on a single line. The default value is 80 corresponding roughly to the width of a standard text document.

The second parameter sets the maximum number of terms per line; a term means a sign, a coefficient, and a name (for example `+ 42 elephants`). The default value is 0, meaning that there is no maximum.

Unnamed Constraints

Reading and writing an LP file with **MOSEK** may change it superficially. If an LP file contains unnamed constraints or objective these are given their generic names when the file is read (however unnamed constraints in **MOSEK** are written without names).

13.2 The MPS File Format

MOSEK supports the standard MPS format with some extensions. For a detailed description of the MPS format see the book by Nazareth *[Naz87]*.

13.2.1 MPS File Structure

The version of the MPS format supported by **MOSEK** allows specification of an optimization problem of the form

$$
\begin{array}{rcccl}
l^c & \leq & Ax + q(x) & \leq & u^c, \\
l^x & \leq & x & \leq & u^x, \\
& & x \in \mathcal{K}, \\
& & x_{\mathcal{J}} & \text{integer},
\end{array}
\tag{13.2}
$$

where

- $x \in \mathbb{R}^n$ is the vector of decision variables.

- $A \in \mathbb{R}^{m \times n}$ is the constraint matrix.

- $l^c \in \mathbb{R}^m$ is the lower limit on the activity for the constraints.

- $u^c \in \mathbb{R}^m$ is the upper limit on the activity for the constraints.

- $l^x \in \mathbb{R}^n$ is the lower limit on the activity for the variables.

- $u^x \in \mathbb{R}^n$ is the upper limit on the activity for the variables.

- $q : \mathbb{R}^n \to \mathbb{R}$ is a vector of quadratic functions. Hence,

$$q_i(x) = \frac{1}{2} x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T.$$

Please note the explicit $\frac{1}{2}$ in the quadratic term and that Q^i is required to be symmetric.

- $\mathcal{K}$ is a convex cone.

- $\mathcal{J} \subseteq \{1, 2, \ldots, n\}$ is an index set of the integer-constrained variables.

An MPS file with one row and one column can be illustrated like this:

```
*         1         2         3         4         5         6
*234567890123456789012345678901234567890123456789012345678901234567890
NAME          [name]
OBJSENSE
[objsense]
OBJNAME
[objname]
ROWS
?  [cname1]
COLUMNS
[vname1]  [cname1]     [value1]      [vname3]  [value2]
RHS
[name]    [cname1]     [value1]      [cname2]  [value2]
RANGES
[name]    [cname1]     [value1]      [cname2]  [value2]
QSECTION      [cname1]
[vname1]  [vname2]     [value1]      [vname3]  [value2]
QMATRIX
[vname1]  [vname2]     [value1]
QUADOBJ
[vname1]  [vname2]     [value1]
QCMATRIX      [cname1]
[vname1]  [vname2]     [value1]
BOUNDS
?? [name]     [vname1]     [value1]
CSECTION      [kname1]     [value1]      [ktype]
[vname1]
ENDATA
```

Here the names in capitals are keywords of the MPS format and names in brackets are custom defined names or values. A couple of notes on the structure:

- Fields: All items surrounded by brackets appear in *fields*. The fields named "valueN" are numerical values. Hence, they must have the format

```
[+|-]XXXXXXX.XXXXXX[[e|E][+|-]XXX]
```

where

```
.. code-block:: text

   X = [0|1|2|3|4|5|6|7|8|9].
```

- Sections: The MPS file consists of several sections where the names in capitals indicate the beginning of a new section. For example, `COLUMNS` denotes the beginning of the columns section.

- Comments: Lines starting with an ∗ are comment lines and are ignored by **MOSEK**.

- Keys: The question marks represent keys to be specified later.

- Extensions: The sections `QSECTION` and `CSECTION` are specific **MOSEK** extensions of the MPS format. The sections `QMATRIX`, `QUADOBJ` and `QCMATRIX` are included for sake of compatibility with other vendors extensions to the MPS format.

The standard MPS format is a fixed format, i.e. everything in the MPS file must be within certain fixed positions. **MOSEK** also supports a *free format*. See Section *13.2.9* for details.

Linear example `lo1.mps`

A concrete example of a MPS file is presented below:

```
* File: lo1.mps
NAME            lo1
OBJSENSE
    MAX
ROWS
 N  obj
 E  c1
 G  c2
 L  c3
COLUMNS
    x1        obj       3
    x1        c1        3
    x1        c2        2
    x2        obj       1
    x2        c1        1
    x2        c2        1
    x2        c3        2
    x3        obj       5
    x3        c1        2
    x3        c2        3
    x4        obj       1
    x4        c2        1
    x4        c3        3
RHS
    rhs       c1        30
    rhs       c2        15
    rhs       c3        25
RANGES
BOUNDS
 UP bound     x2        10
ENDATA
```

Subsequently each individual section in the MPS format is discussed.

Section `NAME`

In this section a name (`[name]`) is assigned to the problem.

OBJSENSE (optional)

This is an optional section that can be used to specify the sense of the objective function. The `OBJSENSE` section contains one line at most which can be one of the following

```
MIN
MINIMIZE
MAX
MAXIMIZE
```

It should be obvious what the implication is of each of these four lines.

OBJNAME (optional)

This is an optional section that can be used to specify the name of the row that is used as objective function. The `OBJNAME` section contains one line at most which has the form

```
objname
```

`objname` should be a valid row name.

ROWS

A record in the `ROWS` section has the form

```
?   [cname1]
```

where the requirements for the fields are as follows:

Field	Starting Position	Max Width	required	Description
?	2	1	Yes	Constraint key
[cname1]	5	8	Yes	Constraint name

Hence, in this section each constraint is assigned an unique name denoted by `[cname1]`. Please note that `[cname1]` starts in position 5 and the field can be at most 8 characters wide. An initial key `?` must be present to specify the type of the constraint. The key can have the values `E`, `G`, `L`, or `N` with the following interpretation:

Constraint type	l_i^c	u_i^c
E	finite	l_i^c
G	finite	∞
L	$-\infty$	finite
N	$-\infty$	∞

In the MPS format an objective vector is not specified explicitly, but one of the constraints having the key `N` will be used as the objective vector c. In general, if multiple `N` type constraints are specified, then the first will be used as the objective vector c.

COLUMNS

In this section the elements of A are specified using one or more records having the form:

```
[vname1]   [cname1]      [value1]        [cname2]   [value2]
```

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

Hence, a record specifies one or two elements a_{ij} of A using the principle that [vname1] and [cname1] determines j and i respectively. Please note that [cname1] must be a constraint name specified in the ROWS section. Finally, [value1] denotes the numerical value of a_{ij}. Another optional element is specified by [cname2], and [value2] for the variable specified by [vname1]. Some important comments are:

- All elements belonging to one variable must be grouped together.

- Zero elements of A should not be specified.

- At least one element for each variable should be specified.

RHS (optional)

A record in this section has the format

```
[name]      [cname1]     [value1]        [cname2]   [value2]
```

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RHS vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The interpretation of a record is that [name] is the name of the RHS vector to be specified. In general, several vectors can be specified. [cname1] denotes a constraint name previously specified in the ROWS section. Now, assume that this name has been assigned to the i th constraint and v_1 denotes the value specified by [value1], then the interpretation of v_1 is:

Constraint type	l_i^c	u_i^c
E	v_1	v_1
G	v_1	
L		v_1
N		

An optional second element is specified by [cname2] and [value2] and is interpreted in the same way. Please note that it is not necessary to specify zero elements, because elements are assumed to be zero.

RANGES (optional)

A record in this section has the form

```
[name]      [cname1]     [value1]        [cname2]   [value2]
```

where the requirements for each fields are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RANGE vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The records in this section are used to modify the bound vectors for the constraints, i.e. the values in l^c and u^c. A record has the following interpretation: [name] is the name of the RANGE vector and [cname1] is a valid constraint name. Assume that [cname1] is assigned to the i th constraint and let v_1 be the value specified by [value1], then a record has the interpretation:

Constraint type	Sign of v_1	l_i^c	u_i^c
E	$-$	$u_i^c + v_1$	
E	$+$		$l_i^c + v_1$
G	$-$ or $+$	$l_i^c + |v_1|$	
L	$-$ or $+$	$u_i^c - |v_1|$	
N			

QSECTION (optional)

Within the QSECTION the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic term belongs. A record in the QSECTION has the form

```
[vname1]   [vname2]     [value1]      [vname3]   [value2]
```

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value
[vname3]	40	8	No	Variable name
[value2]	50	12	No	Numerical value

A record specifies one or two elements in the lower triangular part of the Q^i matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k th and j th variable, then Q_{kj}^i is assigned the value given by [value1] An optional second element is specified in the same way by the fields [vname1], [vname3], and [value2].

The example

$$\begin{aligned}
\text{minimize} \quad & -x_2 + \tfrac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2) \\
\text{subject to} \quad & x_1 + x_2 + x_3 \geq 1, \\
& x \geq 0
\end{aligned}$$

has the following MPS file representation

```
* File: qo1.mps
NAME          qo1
ROWS
N  obj
G  c1
COLUMNS
x1         c1         1.0
x2         obj        -1.0
x2         c1         1.0
x3         c1         1.0
RHS
rhs        c1         1.0
QSECTION      obj
x1         x1         2.0
x1         x3         -1.0
x2         x2         0.2
x3         x3         2.0
ENDATA
```

Regarding the QSECTIONs please note that:

- Only one `QSECTION` is allowed for each constraint.

- The `QSECTION`s can appear in an arbitrary order after the `COLUMNS` section.

- All variable names occurring in the `QSECTION` must already be specified in the `COLUMNS` section.

- All entries specified in a `QSECTION` are assumed to belong to the lower triangular part of the quadratic term of Q .

`QMATRIX/QUADOBJ` (optional)

The `QMATRIX` and `QUADOBJ` sections allow to define the quadratic term of the objective function. They differ in how the quadratic term of the objective function is stored:

- `QMATRIX` It stores all the nonzeros coefficients, withouot taking advantage of the symmetry of the Q matrix.

- `QUADOBJ` It only store the upper diagonal nonzero elements of the Q matrix.

A record in both sections has the form:

```
[vname1]   [vname2]      [value1]
```

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies one elements of the Q matrix in the objective function . Hence, if the names `[vname1]` and `[vname2]` have been assigned to the k th and j th variable, then Q_{kj} is assigned the value given by `[value1]`. Note that a line must apper for each off-diagonal coefficient if using a `QMATRIX` section, while only one entry is required in a `QUADOBJ` section. The quadratic part of the objective function will be evaluated as $1/2 x^T Q x$.

The example

$$
\begin{array}{ll}
\text{minimize} & -x_2 + \frac{1}{2}(2x_1^2 - 2x_1 x_3 + 0.2 x_2^2 + 2x_3^2) \\
\text{subject to} & x_1 + x_2 + x_3 \quad \geq \quad 1, \\
& x \geq 0
\end{array}
$$

has the following MPS file representation using `QMATRIX`

```
* File: qo1_matrix.mps
NAME          qo1_qmatrix
ROWS
 N  obj
 G  c1
COLUMNS
    x1        c1         1.0
    x2        obj        -1.0
    x2        c1         1.0
    x3        c1         1.0
RHS
    rhs       c1         1.0
QMATRIX
    x1        x1         2.0
    x1        x3         -1.0
    x3        x1         -1.0
    x2        x2         0.2
    x3        x3         2.0
ENDATA
```

or the following using `QUADOBJ`

```
* File: qo1_quadobj.mps
NAME            qo1_quadobj
ROWS
 N  obj
 G  c1
COLUMNS
    x1          c1          1.0
    x2          obj         -1.0
    x2          c1          1.0
    x3          c1          1.0
RHS
    rhs         c1          1.0
QUADOBJ
    x1          x1          2.0
    x1          x3          -1.0
    x2          x2          0.2
    x3          x3          2.0
ENDATA
```

Please also note that:

- A `QMATRIX`/`QUADOBJ` section can appear in an arbitrary order after the `COLUMNS` section.

- All variable names occurring in the `QMATRIX`/`QUADOBJ` section must already be specified in the `COLUMNS` section.

13.2.2 `QCMATRIX` (optional)

A `QCMATRIX` section allows to specify the quadratic part of a given constraints. Within the `QCMATRIX` the label `[cname1]` must be a constraint name previously specified in the `ROWS` section. The label `[cname1]` denotes the constraint to which the quadratic term belongs. A record in the `QSECTION` has the form

```
[vname1]  [vname2]    [value1]
```

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies an entry of the Q^i matrix where `[cname1]` specifies the i. Hence, if the names `[vname1]` and `[vname2]` have been assigned to the k th and j th variable, then Q^i_{kj} is assigned the value given by `[value1]`. Moreover, the quadratic term is represented as $1/2x^T Q x$.

The example

$$
\begin{array}{lrcl}
\text{minimize} & x_2 & & \\
\text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\
& \frac{1}{2}(-2x_1 x_3 + 0.2 x_2^2 + 2 x_3^2) & \leq & 10, \\
& x \geq 0
\end{array}
$$

has the following MPS file representation

```
* File: qo1.mps
NAME            qo1
ROWS
 N  obj
 G  c1
 L  q1
COLUMNS
```

```
     x1        c1        1.0
     x2        obj      -1.0
     x2        c1        1.0
     x3        c1        1.0
RHS
     rhs       c1        1.0
     rhs       q1       10.0
QCMATRIX  q1
     x1        x1        2.0
     x1        x3       -1.0
     x3        x1       -1.0
     x2        x2        0.2
     x3        x3        2.0
ENDATA
```

Regarding the `QCMATRIX`s please note that:

- Only one `QCMATRIX` is allowed for each constraint.

- The `QCMATRIX`s can appear in an arbitrary order after the `COLUMNS` section.

- All variable names occurring in the `QSECTION` must already be specified in the `COLUMNS` section.

- A `QCMATRIX` does not exploit the symmetry of Q: an off-diagonal entry (i, j) should apperas twice.

13.2.3 `BOUNDS` (optional)

In the `BOUNDS` section changes to the default bounds vectors l^x and u^x are specified. The default bounds vectors are $l^x = 0$ and $u^x = \infty$. Moreover, it is possible to specify several sets of bound vectors. A record in this section has the form

```
?? [name]    [vname1]    [value1]
```

where the requirements for each field are:

Field	Starting Position	Max Width	Required	Description
??	2	2	Yes	Bound key
[name]	5	8	Yes	Name of the `BOUNDS` vector
[vname1]	15	8	Yes	Variable name
[value1]	25	12	No	Numerical value

Hence, a record in the `BOUNDS` section has the following interpretation:[name] is the name of the bound vector and [vname1] is the name of the variable which bounds are modified by the record. `??` and [value1] are used to modify the bound vectors according to the following table:

??	l^x_j	u^x_j	Made integer (added to $\mathcal{J}$)
FR	$-\infty$	∞	No
FX	v_1	v_1	No
LO	v_1	unchanged	No
MI	$-\infty$	unchanged	No
PL	unchanged	∞	No
UP	unchanged	v_1	No
BV	0	1	Yes
LI	$\lceil v_1 \rceil$	unchanged	Yes
UI	unchanged	$\lfloor v_1 \rfloor$	Yes

v_1 is the value specified by [value1].

13.2.4 CSECTION (optional)

The purpose of the CSECTION is to specify the constraint

$$x \in \mathcal{K}.$$

in (13.2). It is assumed that $\mathcal{K}$ satisfies the following requirements. Let

$$x^t \in \mathbb{R}^{n^t}, \quad t = 1, \dots, k$$

be vectors comprised of parts of the decision variables x so that each decision variable is a member of exactly **one** vector x^t, for example

$$x^1 = \begin{bmatrix} x_1 \\ x_4 \\ x_7 \end{bmatrix} \quad \text{and} \quad x^2 = \begin{bmatrix} x_6 \\ x_5 \\ x_3 \\ x_2 \end{bmatrix}.$$

Next define

$$\mathcal{K} := \left\{ x \in \mathbb{R}^n : \quad x^t \in \mathcal{K}_t, \quad t = 1, \dots, k \right\}$$

where $\mathcal{K}_t$ must have one of the following forms

- $\mathbb{R}$ set:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} \right\}.$$

- Quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : x_1 \geq \sqrt{\sum_{j=2}^{n^t} x_j^2} \right\}. \tag{13.3}$$

- Rotated quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : 2 x_1 x_2 \geq \sum_{j=3}^{n^t} x_j^2, \quad x_1, x_2 \geq 0 \right\}. \tag{13.4}$$

In general, only quadratic and rotated quadratic cones are specified in the MPS file whereas membership of the $\mathbb{R}$ set is not. If a variable is not a member of any other cone then it is assumed to be a member of an $\mathbb{R}$ cone.

Next, let us study an example. Assume that the quadratic cone

$$x_4 \geq \sqrt{x_5^2 + x_8^2}$$

and the rotated quadratic cone

$$x_3 x_7 \geq x_1^2 + x_0^2, \quad x_3, x_7 \geq 0,$$

should be specified in the MPS file. One CSECTION is required for each cone and they are specified as follows:

```
*           1          2          3          4          5          6
*234567890123456789012345678901234567890123456789012345678901234567890
CSECTION       konea       0.0          QUAD
x4
x5
x8
CSECTION       koneb       0.0          RQUAD
x7
x3
x1
x0
```

This first `CSECTION` specifies the cone (13.3) which is given the name `konea`. This is a quadratic cone which is specified by the keyword `QUAD` in the `CSECTION` header. The 0.0 value in the `CSECTION` header is not used by the `QUAD` cone.

The second `CSECTION` specifies the rotated quadratic cone (13.4). Please note the keyword `RQUAD` in the `CSECTION` which is used to specify that the cone is a rotated quadratic cone instead of a quadratic cone. The 0.0 value in the `CSECTION` header is not used by the `RQUAD` cone.

In general, a `CSECTION` header has the format

```
CSECTION       [kname1]    [value1]      [ktype]
```

where the requirement for each field are as follows:

Field	Starting Position	Max Width	Required	Description
[kname1]	5	8	Yes	Name of the cone
[value1]	15	12	No	Cone parameter
[ktype]	25		Yes	Type of the cone.

The possible cone type keys are:

Cone type key	Members	Interpretation.
QUAD	≤ 1	Quadratic cone i.e. (13.3).
RQUAD	≤ 2	Rotated quadratic cone i.e. (13.4).

Please note that a quadratic cone must have at least one member whereas a rotated quadratic cone must have at least two members. A record in the `CSECTION` has the format

```
[vname1]
```

where the requirements for each field are

Field	Starting Position	Max Width	required	Description
[vname1]	2	8	Yes	A valid variable name

The most important restriction with respect to the `CSECTION` is that a variable must occur in only one `CSECTION`.

13.2.5 `ENDATA`

This keyword denotes the end of the MPS file.

13.2.6 Integer Variables

Using special bound keys in the `BOUNDS` section it is possible to specify that some or all of the variables should be integer-constrained i.e. be members of $\mathcal{J}$. However, an alternative method is available.

This method is available only for backward compatibility and we recommend that it is not used. This method requires that markers are placed in the `COLUMNS` section as in the example:

```
COLUMNS
x1        obj        -10.0          c1        0.7
x1        c2         0.5            c3        1.0
x1        c4         0.1
* Start of integer-constrained variables.
MARK000   'MARKER'                  'INTORG'
x2        obj        -9.0           c1        1.0
x2        c2         0.8333333333   c3        0.66666667
x2        c4         0.25
x3        obj        1.0            c6        2.0
MARK001   'MARKER'                  'INTEND'
```

- End of integer-constrained variables.

Please note that special marker lines are used to indicate the start and the end of the integer variables. Furthermore be aware of the following

- IMPORTANT: All variables between the markers are assigned a default lower bound of 0 and a default upper bound of 1. **This may not be what is intended.** If it is not intended, the correct bounds should be defined in the BOUNDS section of the MPS formatted file.

- **MOSEK** ignores field 1, i.e. MARK0001 and MARK001, however, other optimization systems require them.

- Field 2, i.e. MARKER, must be specified including the single quotes. This implies that no row can be assigned the name MARKER.

- Field 3 is ignored and should be left blank.

- Field 4, i.e. INTORG and INTEND, must be specified.

- It is possible to specify several such integer marker sections within the COLUMNS section.

13.2.7 General Limitations

- An MPS file should be an ASCII file.

13.2.8 Interpretation of the MPS Format

Several issues related to the MPS format are not well-defined by the industry standard. However, **MOSEK** uses the following interpretation:

- If a matrix element in the COLUMNS section is specified multiple times, then the multiple entries are added together.

- If a matrix element in a QSECTION section is specified multiple times, then the multiple entries are added together.

13.2.9 The Free MPS Format

MOSEK supports a free format variation of the MPS format. The free format is similar to the MPS file format but less restrictive, e.g. it allows longer names. However, it also presents two main limitations:

- A name must not contain any blanks.

- By default a line in the MPS file must not contain more than 1024 characters. However, by modifying the parameter *MSK_IPAR_READ_MPS_WIDTH* an arbitrary large line width will be accepted.

To use the free MPS format instead of the default MPS format the **MOSEK** parameter *MSK_IPAR_READ_MPS_FORMAT* should be changed.

13.3 The OPF Format

The *Optimization Problem Format (OPF)* is an alternative to LP and MPS files for specifying optimization problems. It is row-oriented, inspired by the CPLEX LP format.

Apart from containing objective, constraints, bounds etc. it may contain complete or partial solutions, comments and extra information relevant for solving the problem. It is designed to be easily read and modified by hand and to be forward compatible with possible future extensions.

Intended use

The OPF file format is meant to replace several other files:

- The LP file format: Any problem that can be written as an LP file can be written as an OPF file too; furthermore it naturally accommodates ranged constraints and variables as well as arbitrary characters in names, fixed expressions in the objective, empty constraints, and conic constraints.

- Parameter files: It is possible to specify integer, double and string parameters along with the problem (or in a separate OPF file).

- Solution files: It is possible to store a full or a partial solution in an OPF file and later reload it.

13.3.1 The File Format

The format uses tags to structure data. A simple example with the basic sections may look like this:

```
[comment]
This is a comment. You may write almost anything here...
[/comment]

# This is a single-line comment.

[objective min 'myobj']
x + 3 y + x^2 + 3 y^2 + z + 1
[/objective]

[constraints]
[con 'con01'] 4 <= x + y  [/con]
[/constraints]

[bounds]
[b] -10 <= x,y <= 10   [/b]

[cone quad] x,y,z [/cone]
[/bounds]
```

A scope is opened by a tag of the form `[tag]` and closed by a tag of the form `[/tag]`. An opening tag may accept a list of unnamed and named arguments, for examples:

```
[tag value] tag with one unnamed argument [/tag]
[tag arg=value] tag with one named argument in quotes [/tag]
```

Unnamed arguments are identified by their order, while named arguments may appear in any order, but never before an unnamed argument. The `value` can be a quoted, single-quoted or double-quoted text string, i.e.

```
[tag 'value']     single-quoted value [/tag]
[tag arg='value'] single-quoted value [/tag]
[tag "value"]     double-quoted value [/tag]
[tag arg="value"] double-quoted value [/tag]
```

Sections

The recognized tags are

[comment]

A comment section. This can contain *almost* any text: Between single quotes (') or double quotes (") any text may appear. Outside quotes the markup characters ([and]) must be prefixed by backslashes. Both single and double quotes may appear alone or inside a pair of quotes if it is prefixed by a backslash.

[objective]

The objective function: This accepts one or two parameters, where the first one (in the above example `min`) is either `min` or `max` (regardless of case) and defines the objective sense, and the second one (above `myobj`), if present, is the objective name. The section may contain linear and quadratic expressions. If several objectives are specified, all but the last are ignored.

[constraints]

This does not directly contain any data, but may contain the subsection `con` defining a linear constraint.

[con] defines a single constraint; if an argument is present ([con NAME]) this is used as the name of the constraint, otherwise it is given a null-name. The section contains a constraint definition written as linear and quadratic expressions with a lower bound, an upper bound, with both or with an equality. Examples:

```
[constraints]
[con 'con1'] 0 <= x + y        [/con]
[con 'con2'] 0 >= x + y        [/con]
[con 'con3'] 0 <= x + y <= 10 [/con]
[con 'con4']     x + y  = 10 [/con]
[/constraints]
```

Constraint names are unique. If a constraint is specified which has the same name as a previously defined constraint, the new constraint replaces the existing one.

[bounds]

This does not directly contain any data, but may contain the subsections `b` (linear bounds on variables) and `cone` (quadratic cone).

[b]. Bound definition on one or several variables separated by comma (,). An upper or lower bound on a variable replaces any earlier defined bound on that variable. If only one bound (upper or lower) is given only this bound is replaced. This means that upper and lower bounds can be specified separately. So the OPF bound definition:

```
[b]  x,y >= -10  [/b]
[b]  x,y <= 10   [/b]
```

results in the bound $-10 \le x, y \le 10$.

[cone]. currently supports the *quadratic cone* and the *rotated quadratic cone*.

A conic constraint is defined as a set of variables which belong to a single unique cone.

- A quadratic cone of n variables $x_1, \ldots, x_n$ defines a constraint of the form

$$x_1^2 \geq \sum_{i=2}^{n} x_i^2, \quad x_1 \geq 0.$$

- A rotated quadratic cone of n variables $x_1, \ldots, x_n$ defines a constraint of the form

$$2x_1 x_2 \geq \sum_{i=3}^{n} x_i^2, \quad x_1, x_2 \geq 0.$$

A [bounds]-section example:

```
[bounds]
[b]  0 <= x,y <= 10  [/b] # ranged bound
[b] 10 >= x,y >=  0  [/b] # ranged bound
[b]  0 <= x,y <= inf [/b] # using inf
[b]       x,y free   [/b] # free variables
# Let (x,y,z,w) belong to the cone K
[cone quad]  x,y,z,w  [/cone] # quadratic cone
[cone rquad] x,y,z,w  [/cone] # rotated quadratic cone
[/bounds]
```

By default all variables are free.

[variables]

This defines an ordering of variables as they should appear in the problem. This is simply a space-separated list of variable names. Optionally, an attribute can be added [variables disallow_new_variables] indicating that if any variable not listed here occurs later in the file it is an error.

[integer]

This contains a space-separated list of variables and defines the constraint that the listed variables must be integer values.

[hints]

This may contain only non-essential data; for example estimates of the number of variables, constraints and non-zeros. Placed before all other sections containing data this may reduce the time spent reading the file.

In the hints section, any subsection which is not recognized by **MOSEK** is simply ignored. In this section a hint in a subsection is defined as follows:

```
[hint ITEM] value [/hint]
```

where ITEM may be replaced by numvar (number of variables), numcon (number of linear/quadratic constraints), numanz (number of linear non-zeros in constraints) and numqnz (number of quadratic non-zeros in constraints).

[solutions]

This section can contain a set of full or partial solutions to a problem. Each solution must be specified using a [solution]-section, i.e.

```
[solutions]
[solution]...[/solution] #solution 1
[solution]...[/solution] #solution 2
#other solutions....
[solution]...[/solution] #solution n
[/solutions]
```

Note that a [solution]-section must be always specified inside a [solutions]-section. The syntax of a [solution]-section is the following:

```
[solution SOLTYPE status=STATUS]...[/solution]
```

where SOLTYPE is one of the strings

- interior, a non-basic solution,

- basic, a basic solution,

- integer, an integer solution,

and STATUS is one of the strings

- UNKNOWN,

- OPTIMAL,

- INTEGER_OPTIMAL,

- PRIM_FEAS,

- DUAL_FEAS,

- PRIM_AND_DUAL_FEAS,

- NEAR_OPTIMAL,

- NEAR_PRIM_FEAS,

- NEAR_DUAL_FEAS,

- NEAR_PRIM_AND_DUAL_FEAS,

- PRIM_INFEAS_CER,

- DUAL_INFEAS_CER,

- NEAR_PRIM_INFEAS_CER,

- NEAR_DUAL_INFEAS_CER,

- NEAR_INTEGER_OPTIMAL.

Most of these values are irrelevant for input solutions; when constructing a solution for simplex hot-start or an initial solution for a mixed integer problem the safe setting is UNKNOWN.

A [solution]-section contains [con] and [var] sections. Each [con] and [var] section defines solution information for a single variable or constraint, specified as list of KEYWORD/value pairs, in any order, written as

```
KEYWORD=value
```

Allowed keywords are as follows:

- sk. The status of the item, where the value is one of the following strings:

  - LOW, the item is on its lower bound.

- UPR, the item is on its upper bound.

- FIX, it is a fixed item.

- BAS, the item is in the basis.

- SUPBAS, the item is super basic.

- UNK, the status is unknown.

- INF, the item is outside its bounds (infeasible).

- lvl Defines the level of the item.

- sl Defines the level of the dual variable associated with its lower bound.

- su Defines the level of the dual variable associated with its upper bound.

- sn Defines the level of the variable associated with its cone.

- y Defines the level of the corresponding dual variable (for constraints only).

A [var] section should always contain the items sk, lvl, sl and su. Items sl and su are not required for integer solutions.

A [con] section should always contain sk, lvl, sl, su and y.

An example of a solution section

```
[solution basic status=UNKNOWN]
[var x0] sk=LOW    lvl=5.0        [/var]
[var x1] sk=UPR    lvl=10.0       [/var]
[var x2] sk=SUPBAS lvl=2.0  sl=1.5 su=0.0 [/var]

[con c0] sk=LOW    lvl=3.0 y=0.0 [/con]
[con c0] sk=UPR    lvl=0.0 y=5.0 [/con]
[/solution]
```

- [vendor] This contains solver/vendor specific data. It accepts one argument, which is a vendor ID – for **MOSEK** the ID is simply mosek – and the section contains the subsection parameters defining solver parameters. When reading a vendor section, any unknown vendor can be safely ignored. This is described later.

Comments using the # may appear anywhere in the file. Between the # and the following line-break any text may be written, including markup characters.

Numbers

Numbers, when used for parameter values or coefficients, are written in the usual way by the printf function. That is, they may be prefixed by a sign (+ or -) and may contain an integer part, decimal part and an exponent. The decimal point is always . (a dot). Some examples are

```
1
1.0
.0
1.
1e10
1e+10
1e-10
```

Some *invalid* examples are

```
e10   # invalid, must contain either integer or decimal part
.     # invalid
.e10  # invalid
```

More formally, the following standard regular expression describes numbers as used:

```
[+|-]?([0-9]+[.][0-9]*|[.][0-9]+)([eE][+|-]?[0-9]+)?
```

Names

Variable names, constraint names and objective name may contain arbitrary characters, which in some cases must be enclosed by quotes (single or double) that in turn must be preceded by a backslash. Unquoted names must begin with a letter (`a-z` or `A-Z`) and contain only the following characters: the letters `a-z` and `A-Z`, the digits `0-9`, braces (`{` and `}`) and underscore (`_`).

Some examples of legal names:

```
an_unquoted_name
another_name{123}
'single quoted name'
"double quoted name"
"name with \\"quote\\" in it"
"name with []s in it"
```

13.3.2 Parameters Section

In the `vendor` section solver parameters are defined inside the `parameters` subsection. Each parameter is written as

```
[p PARAMETER_NAME] value [/p]
```

where `PARAMETER_NAME` is replaced by a **MOSEK** parameter name, usually of the form `MSK_IPAR_...`, `MSK_DPAR_...` or `MSK_SPAR_...`, and the `value` is replaced by the value of that parameter; both integer values and named values may be used. Some simple examples are

```
[vendor mosek]
[parameters]
[p MSK_IPAR_OPF_MAX_TERMS_PER_LINE] 10      [/p]
[p MSK_IPAR_OPF_WRITE_PARAMETERS]   MSK_ON [/p]
[p MSK_DPAR_DATA_TOL_BOUND_INF]     1.0e18 [/p]
[/parameters]
[/vendor]
```

13.3.3 Writing OPF Files from MOSEK

To write an OPF file set the parameter *MSK_IPAR_WRITE_DATA_FORMAT* to *"MSK_DATA_FORMAT_OP"* as this ensures that OPF format is used.

Then modify the following parameters to define what the file should contain:

MSK_IPAR_OPF_WRITE_SOL_BAS	Include basic solution, if defined.
MSK_IPAR_OPF_WRITE_SOL_ITG	Include integer solution, if defined.
MSK_IPAR_OPF_WRITE_SOL_ITR	Include interior solution, if defined.
MSK_IPAR_OPF_WRITE_SOLUTIONS	Include solutions if they are defined. If this is off, no solutions are included.
MSK_IPAR_OPF_WRITE_HEADER	Include a small header with comments.
MSK_IPAR_OPF_WRITE_PROBLEM	Include the problem itself — objective, constraints and bounds.
MSK_IPAR_OPF_WRITE_PARAMETERS	Include all parameter settings.
MSK_IPAR_OPF_WRITE_HINTS	Include hints about the size of the problem.

13.3.4 Examples

This section contains a set of small examples written in OPF and describing how to formulate linear, quadratic and conic problems.

Linear Example `lo1.opf`

Consider the example:

$$
\begin{array}{rrrrrrrrrrl}
\text{maximize} & 3x_0 & + & 1x_1 & + & 5x_2 & + & 1x_3 & & \\
\text{subject to} & 3x_0 & + & 1x_1 & + & 2x_2 & & & = & 30, \\
& 2x_0 & + & 1x_1 & + & 3x_2 & + & 1x_3 & \geq & 15, \\
& & & 2x_1 & & & + & 3x_3 & \leq & 25,
\end{array}
$$

having the bounds

$$
\begin{array}{rclcl}
0 & \leq & x_0 & \leq & \infty, \\
0 & \leq & x_1 & \leq & 10, \\
0 & \leq & x_2 & \leq & \infty, \\
0 & \leq & x_3 & \leq & \infty.
\end{array}
$$

In the `OPF` format the example is displayed as shown in Listing 13.1.

Listing 13.1: Example of an OPF file for a linear problem.

```
[comment]
  The lo1 example in OPF format
[/comment]

[hints]
  [hint NUMVAR] 4 [/hint]
  [hint NUMCON] 3 [/hint]
  [hint NUMANZ] 9 [/hint]
[/hints]

[variables disallow_new_variables]
  x1 x2 x3 x4
[/variables]

[objective maximize 'obj']
   3 x1 + x2 + 5 x3 + x4
[/objective]

[constraints]
  [con 'c1'] 3 x1 +   x2 + 2 x3          = 30 [/con]
  [con 'c2'] 2 x1 +   x2 + 3 x3 +   x4 >= 15 [/con]
  [con 'c3']         2 x2        + 3 x4 <= 25 [/con]
[/constraints]

[bounds]
  [b] 0 <= * [/b]
  [b] 0 <= x2 <= 10 [/b]
[/bounds]
```

Quadratic Example `qo1.opf`

An example of a quadratic optimization problem is

$$
\begin{array}{rcl}
\text{minimize} & & x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2 \\
\text{subject to} \quad 1 & \leq & x_1 + x_2 + x_3, \\
& & x \geq 0.
\end{array}
$$

This can be formulated in `opf` as shown below.

Listing 13.2: Example of an OPF file for a quadratic problem.

```
[comment]
  The qo1 example in OPF format
[/comment]

[hints]
  [hint NUMVAR] 3 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
  [hint NUMQNZ] 4 [/hint]
[/hints]

[variables disallow_new_variables]
  x1 x2 x3
[/variables]

[objective minimize 'obj']
  # The quadratic terms are often written with a factor of 1/2 as here,
  # but this is not required.

  - x2 + 0.5 ( 2.0 x1 ^ 2 - 2.0 x3 * x1 + 0.2 x2 ^ 2 + 2.0 x3 ^ 2 )
[/objective]

[constraints]
  [con 'c1'] 1.0 <= x1 + x2 + x3 [/con]
[/constraints]

[bounds]
  [b] 0 <= * [/b]
[/bounds]
```

Conic Quadratic Example `cqo1.opf`

Consider the example:

$$
\begin{array}{rlcl}
\text{minimize} & x_3 + x_4 + x_5 \\
\text{subject to} & x_0 + x_1 + 2x_2 & = & 1, \\
& x_0, x_1, x_2 & \geq & 0, \\
& x_3 \geq \sqrt{x_0^2 + x_1^2}, \\
& 2x_4 x_5 \geq x_2^2.
\end{array}
$$

Please note that the type of the cones is defined by the parameter to `[cone ...]`; the content of the `cone`-section is the names of variables that belong to the cone. The resulting OPF file is in Listing 13.3.

Listing 13.3: Example of an OPF file for a conic quadratic problem.

```
[comment]
  The cqo1 example in OPF format.
[/comment]

[hints]
  [hint NUMVAR] 6 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
[/hints]

[variables disallow_new_variables]
  x1 x2 x3 x4 x5 x6
[/variables]
```

```
[objective minimize 'obj']
   x4 + x5 + x6
[/objective]

[constraints]
  [con 'c1']  x1 + x2 + 2e+00 x3 = 1e+00 [/con]
[/constraints]

[bounds]
  # We let all variables default to the positive orthant
  [b] 0 <= * [/b]

  # ...and change those that differ from the default
  [b] x4,x5,x6 free [/b]

  # Define quadratic cone: x4 >= sqrt( x1^2 + x2^2 )
  [cone quad 'k1'] x4, x1, x2 [/cone]

  # Define rotated quadratic cone: 2 x5 x6 >= x3^2
  [cone rquad 'k2'] x5, x6, x3 [/cone]
[/bounds]
```

Mixed Integer Example `milo1.opf`

Consider the mixed integer problem:

$$
\begin{array}{rrcl}
\text{maximize} & x_0 + 0.64 x_1 & & \\
\text{subject to} & 50 x_0 + 31 x_1 & \leq & 250, \\
& 3 x_0 - 2 x_1 & \geq & -4, \\
& x_0, x_1 \geq 0 & & \text{and integer}
\end{array}
$$

This can be implemented in OPF with the file in Listing 13.4.

Listing 13.4: Example of an OPF file for a mixed-integer linear problem.

```
[comment]
  The milo1 example in OPF format
[/comment]

[hints]
  [hint NUMVAR] 2 [/hint]
  [hint NUMCON] 2 [/hint]
  [hint NUMANZ] 4 [/hint]
[/hints]

[variables disallow_new_variables]
  x1 x2
[/variables]

[objective maximize 'obj']
   x1 + 6.4e-1 x2
[/objective]

[constraints]
  [con 'c1'] 5e+1 x1 + 3.1e+1 x2 <= 2.5e+2 [/con]
  [con 'c2'] -4 <= 3 x1 - 2 x2 [/con]
[/constraints]

[bounds]
  [b] 0 <= * [/b]
[/bounds]
```

```
[integer]
  x1 x2
[/integer]
```

13.4 The CBF Format

This document constitutes the technical reference manual of the *Conic Benchmark Format* with file extension: `.cbf` or `.CBF`. It unifies linear, second-order cone (also known as conic quadratic) and semidefinite optimization with mixed-integer variables. The format has been designed with benchmark libraries in mind, and therefore focuses on compact and easily parsable representations. The problem structure is separated from the problem data, and the format moreover facilitates benchmarking of hotstart capability through sequences of changes.

13.4.1 How Instances Are Specified

This section defines the spectrum of conic optimization problems that can be formulated in terms of the keywords of the CBF format.

In the CBF format, conic optimization problems are considered in the following form:

$$
\begin{aligned}
\min / \max \quad & g^{obj} \\
\text{s.t.} \quad & g_i \in \mathcal{K}_i, && i \in \mathcal{I}, \\
& G_i \in \mathcal{K}_i, && i \in \mathcal{I}^{PSD}, \\
& x_j \in \mathcal{K}_j, && j \in \mathcal{J}, \\
& \overline{X}_j \in \mathcal{K}_j, && j \in \mathcal{J}^{PSD}.
\end{aligned}
\tag{13.5}
$$

- **Variables** are either scalar variables, x_j for $j \in \mathcal{J}$, or variables, $\overline{X}_j$ for $j \in \mathcal{J}^{PSD}$. Scalar variables can also be declared as integer.

- **Constraints** are affine expressions of the variables, either scalar-valued g_i for $i \in \mathcal{I}$, or matrix-valued G_i for $i \in \mathcal{I}^{PSD}$

$$
g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,
$$

$$
G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i.
$$

- The **objective function** is a scalar-valued affine expression of the variables, either to be minimized or maximized. We refer to this expression as g^{obj}

$$
g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj}.
$$

CBF format can represent the following cones $\mathcal{K}$:

- **Free domain** - A cone in the linear family defined by

$$
\{x \in \mathbb{R}^n\}, \text{ for } n \geq 1.
$$

- **Positive orthant** - A cone in the linear family defined by

$$
\{x \in \mathbb{R}^n \mid x_j \geq 0 \text{ for } j = 1, \ldots, n\}, \text{ for } n \geq 1.
$$

- **Negative orthant** - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_j \leq 0 \text{ for } j = 1, \ldots, n\}, \text{ for } n \geq 1.$$

- **Fixpoint zero** - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_j = 0 \text{ for } j = 1, \ldots, n\}, \text{ for } n \geq 1.$$

- **Quadratic cone** - A cone in the second-order cone family defined by

$$\left\{ \begin{pmatrix} p \\ x \end{pmatrix} \in \mathbb{R} \times \mathbb{R}^{n-1}, \ p^2 \geq x^T x, \ p \geq 0 \right\}, \text{ for } n \geq 2.$$

- **Rotated quadratic cone** - A cone in the second-order cone family defined by

$$\left\{ \begin{pmatrix} p \\ q \\ x \end{pmatrix} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{n-2}, \ 2pq \geq x^T x, \ p \geq 0, \ q \geq 0 \right\}, \text{ for } n \geq 3.$$

13.4.2 The Structure of CBF Files

This section defines how information is written in the CBF format, without being specific about the type of information being communicated.

All information items belong to exactly one of the three groups of information. These information groups, and the order they must appear in, are:

1. File format.
2. Problem structure.
3. Problem data.

The first group, file format, provides information on how to interpret the file. The second group, problem structure, provides the information needed to deduce the type and size of the problem instance. Finally, the third group, problem data, specifies the coefficients and constants of the problem instance.

Information items

The format is composed as a list of information items. The first line of an information item is the `KEYWORD`, revealing the type of information provided. The second line - of some keywords only - is the `HEADER`, typically revealing the size of information that follows. The remaining lines are the `BODY` holding the actual information to be specified.

```
KEYWORD
BODY

KEYWORD
HEADER
BODY
```

The `KEYWORD` determines how each line in the `HEADER` and `BODY` is structured. Moreover, the number of lines in the `BODY` follows either from the `KEYWORD`, the `HEADER`, or from another information item required to precede it.

Embedded hotstart-sequences

A sequence of problem instances, based on the same problem structure, is within a single file. This is facilitated via the `CHANGE` within the problem data information group, as a separator between the information items of each instance. The information items following a `CHANGE` keyword are appending to, or changing (e.g., setting coefficients back to their default value of zero), the problem data of the preceding instance.

The sequence is intended for benchmarking of hotstart capability, where the solvers can reuse their internal state and solution (subject to the achieved accuracy) as warmpoint for the succeeding instance. Whenever this feature is unsupported or undesired, the keyword `CHANGE` should be interpreted as the end of file.

File encoding and line width restrictions

The format is based on the US-ASCII printable character set with two extensions as listed below. Note, by definition, that none of these extensions can be misinterpreted as printable US-ASCII characters:

- A line feed marks the end of a line, carriage returns are ignored.
- Comment-lines may contain unicode characters in UTF-8 encoding.

The line width is restricted to 512 bytes, with 3 bytes reserved for the potential carriage return, line feed and null-terminator.

Integers and floating point numbers must follow the ISO C decimal string representation in the standard C locale. The format does not impose restrictions on the magnitude of, or number of significant digits in numeric data, but the use of 64-bit integers and 64-bit IEEE 754 floating point numbers should be sufficient to avoid loss of precision.

Comment-line and whitespace rules

The format allows single-line comments respecting the following rule:

- Lines having first byte equal to '#' (US-ASCII 35) are comments, and should be ignored. Comments are only allowed between information items.

Given that a line is not a comment-line, whitespace characters should be handled according to the following rules:

- Leading and trailing whitespace characters should be ignored.
  - The seperator between multiple pieces of information on one line, is either one or more whitespace characters.
- Lines containing only whitespace characters are empty, and should be ignored. Empty lines are only allowed between information items.

13.4.3 Problem Specification

The problem structure

The problem structure defines the objective sense, whether it is minimization and maximization. It also defines the index sets, $\mathcal{J}$, $\mathcal{J}^{PSD}$, $\mathcal{I}$ and $\mathcal{I}^{PSD}$, which are all numbered from zero, $\{0, 1, \ldots\}$, and empty until explicitly constructed.

- **Scalar variables** are constructed in vectors restricted to a conic domain, such as $(x_0, x_1) \in \mathbb{R}_+^2$, $(x_2, x_3, x_4) \in \mathcal{Q}^3$, etc. In terms of the Cartesian product, this generalizes to

$$x \in \mathcal{K}_1^{n_1} \times \mathcal{K}_2^{n_2} \times \cdots \times \mathcal{K}_k^{n_k}$$

which in the CBF format becomes:

```
VAR
n k
K1 n1
K2 n2
...
Kk nk
```

where $\sum_i n_i = n$ is the total number of scalar variables. The list of supported cones is found in Table 13.3. Integrality of scalar variables can be specified afterwards.

- **PSD variables** are constructed one-by-one. That is, $X_j \succeq \mathbf{0}^{n_j \times n_j}$ for $j \in \mathcal{J}^{PSD}$, constructs a matrix-valued variable of size $n_j \times n_j$ restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes:

```
PSDVAR
N
n1
n2
...
nN
```

where N is the total number of PSD variables.

- **Scalar constraints** are constructed in vectors restricted to a conic domain, such as $(g_0, g_1) \in \mathbb{R}_+^2$, $(g_2, g_3, g_4) \in \mathcal{Q}^3$, etc. In terms of the Cartesian product, this generalizes to

$$g \in \mathcal{K}_1^{m_1} \times \mathcal{K}_2^{m_2} \times \cdots \times \mathcal{K}_k^{m_k}$$

which in the CBF format becomes:

```
CON
m k
K1 m1
K2 m2
..
Kk mk
```

where $\sum_i m_i = m$ is the total number of scalar constraints. The list of supported cones is found in Table 13.3.

- **PSD constraints** are constructed one-by-one. That is, $G_i \succeq \mathbf{0}^{m_i \times m_i}$ for $i \in \mathcal{I}^{PSD}$, constructs a matrix-valued affine expressions of size $m_i \times m_i$ restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes

```
PSDCON
M
m1
m2
..
mM
```

where M is the total number of PSD constraints.

With the objective sense, variables (with integer indications) and constraints, the definitions of the many affine expressions follow in problem data.

Problem data

The problem data defines the coefficients and constants of the affine expressions of the problem instance. These are considered zero until explicitly defined, implying that instances with no keywords from this

information group are, in fact, valid. Duplicating or conflicting information is a failure to comply with the standard. Consequently, two coefficients written to the same position in a matrix (or to transposed positions in a symmetric matrix) is an error.

The affine expressions of the objective, g^{obj}, of the scalar constraints, g_i, and of the PSD constraints, G_i, are defined separately. The following notation uses the standard trace inner product for matrices, $\langle X, Y \rangle = \sum_{i,j} X_{ij} Y_{ij}$.

- The affine expression of the objective is defined as

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj},$$

in terms of the symmetric matrices, F_j^{obj}, and scalars, a_j^{obj} and b^{obj}.

- The affine expressions of the scalar constraints are defined, for $i \in \mathcal{I}$, as

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$

in terms of the symmetric matrices, F_{ij}, and scalars, a_{ij} and b_i.

- The affine expressions of the PSD constraints are defined, for $i \in \mathcal{I}^{PSD}$, as

$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i,$$

in terms of the symmetric matrices, H_{ij} and D_i.

List of cones

The format uses an explicit syntax for symmetric positive semidefinite cones as shown above. For scalar variables and constraints, constructed in vectors, the supported conic domains and their minimum sizes are given as follows.

Table 13.3: Cones available in the CBF format

Name	CBF keyword	Cone family
Free domain	F	linear
Positive orthant	L+	linear
Negative orthant	L-	linear
Fixpoint zero	L=	linear
Quadratic cone	Q	second-order
Rotated quadratic cone	QR	second-order

13.4.4 File Format Keywords

VER

Description: The version of the Conic Benchmark Format used to write the file.

`HEADER`: None

`BODY`: One line formatted as:

```
INT
```

This is the version number.

Must appear exactly once in a file, as the first keyword.

OBJSENSE

Description: Define the objective sense.

`HEADER`: None

`BODY`: One line formatted as:

```
STR
```

having `MIN` indicates minimize, and `MAX` indicates maximize. Capital letters are required.

Must appear exactly once in a file.

PSDVAR

Description: Construct the PSD variables.

`HEADER`: One line formatted as:

```
INT
```

This is the number of PSD variables in the problem.

`BODY`: A list of lines formatted as:

```
INT
```

This indicates the number of rows (equal to the number of columns) in the matrix-valued PSD variable. The number of lines should match the number stated in the header.

VAR

Description: Construct the scalar variables.

`HEADER`: One line formatted as:

```
INT INT
```

This is the number of scalar variables, followed by the number of conic domains they are restricted to.

`BODY`: A list of lines formatted as:

```
STR INT
```

This indicates the cone name (see Table 13.3), and the number of scalar variables restricted to this cone. These numbers should add up to the number of scalar variables stated first in the header. The number of lines should match the second number stated in the header.

INT

Description: Declare integer requirements on a selected subset of scalar variables.

`HEADER`: one line formatted as:

```
INT
```

This is the number of integer scalar variables in the problem.

`BODY`: a list of lines formatted as:

```
INT
```

This indicates the scalar variable index $j \in \mathcal{J}$. The number of lines should match the number stated in the header.

Can only be used after the keyword `VAR`.

PSDCON

Description: Construct the PSD constraints.

`HEADER`: One line formatted as:

```
INT
```

This is the number of PSD constraints in the problem.

`BODY`: A list of lines formatted as:

```
INT
```

This indicates the number of rows (equal to the number of columns) in the matrix-valued affine expression of the PSD constraint. The number of lines should match the number stated in the header.

Can only be used after these keywords: `PSDVAR`, `VAR`.

CON

Description: Construct the scalar constraints.

`HEADER`: One line formatted as:

```
INT INT
```

This is the number of scalar constraints, followed by the number of conic domains they restrict to.

`BODY`: A list of lines formatted as:

```
STR INT
```

This indicates the cone name (see Table 13.3), and the number of affine expressions restricted to this cone. These numbers should add up to the number of scalar constraints stated first in the header. The number of lines should match the second number stated in the header.

Can only be used after these keywords: `PSDVAR`, `VAR`

OBJFCOORD

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices F_j^{obj}, as used in the objective.

`HEADER`: One line formatted as:

```
INT
```

This is the number of coordinates to be specified.

`BODY`: A list of lines formatted as:

```
INT INT INT REAL
```

This indicates the PSD variable index $j \in \mathcal{J}^{PSD}$, the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

OBJACOORD

Description: Input sparse coordinates (pairs) to define the scalars, a_j^{obj}, as used in the objective.

HEADER: One line formatted as:

```
INT
```

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

```
INT REAL
```

This indicates the scalar variable index $j \in \mathcal{J}$ and the coefficient value. The number of lines should match the number stated in the header.

OBJBCOORD

Description: Input the scalar, b^{obj}, as used in the objective.

HEADER: None.

BODY: One line formatted as:

```
REAL
```

This indicates the coefficient value.

FCOORD

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices, F_{ij}, as used in the scalar constraints.

HEADER: One line formatted as:

```
INT
```

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

```
INT INT INT INT REAL
```

This indicates the scalar constraint index $i \in \mathcal{I}$, the PSD variable index $j \in \mathcal{J}^{PSD}$, the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

ACOORD

Description: Input sparse coordinates (triplets) to define the scalars, a_{ij}, as used in the scalar constraints.

HEADER: One line formatted as:

```
INT
```

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

```
INT INT REAL
```

This indicates the scalar constraint index $i \in \mathcal{I}$, the scalar variable index $j \in \mathcal{J}$ and the coefficient value. The number of lines should match the number stated in the header.

BCOORD

Description: Input sparse coordinates (pairs) to define the scalars, b_i, as used in the scalar constraints.

HEADER: One line formatted as:

```
INT
```

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

```
INT REAL
```

This indicates the scalar constraint index $i \in \mathcal{I}$ and the coefficient value. The number of lines should match the number stated in the header.

HCOORD

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices, H_{ij}, as used in the PSD constraints.

HEADER: One line formatted as:

```
INT
```

This is the number of coordinates to be specified.

BODY: A list of lines formatted as

```
INT INT INT INT REAL
```

This indicates the PSD constraint index $i \in \mathcal{I}^{PSD}$, the scalar variable index $j \in \mathcal{J}$, the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

DCOORD

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices, D_i, as used in the PSD constraints.

HEADER: One line formatted as

```
INT
```

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

```
INT INT INT REAL
```

This indicates the PSD constraint index $i \in \mathcal{I}^{PSD}$, the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

CHANGE

Start of a new instance specification based on changes to the previous. Can be interpreted as the end of file when the hotstart-sequence is unsupported or undesired.

`BODY`: None

`Header`: None

13.4.5 CBF Format Examples

Minimal Working Example

The conic optimization problem (13.6) , has three variables in a quadratic cone - first one is integer - and an affine expression in domain 0 (equality constraint).

$$
\begin{array}{ll}
\text{minimize} & 5.1\,x_0 \\
\text{subject to} & 6.2\,x_1 + 7.3\,x_2 - 8.4 \in \{0\} \\
& x \in \mathcal{Q}^3,\ x_0 \in \mathbb{Z}.
\end{array}
\tag{13.6}
$$

Its formulation in the Conic Benchmark Format begins with the version of the CBF format used, to safeguard against later revisions.

```
VER
1
```

Next follows the problem structure, consisting of the objective sense, the number and domain of variables, the indices of integer variables, and the number and domain of scalar-valued affine expressions (i.e., the equality constraint).

```
OBJSENSE
MIN

VAR
3 1
Q 3

INT
1
0

CON
1 1
L= 1
```

Finally follows the problem data, consisting of the coefficients of the objective, the coefficients of the constraints, and the constant terms of the constraints. All data is specified on a sparse coordinate form.

```
OBJACOORD
1
0 5.1

ACOORD
2
0 1 6.2
0 2 7.3

BCOORD
1
0 -8.4
```

This concludes the example.

Mixing Linear, Second-order and Semidefinite Cones

The conic optimization problem (13.7), has a semidefinite cone, a quadratic cone over unordered subindices, and two equality constraints.

$$
\begin{aligned}
\text{minimize} \quad & \left\langle \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}, X_1 \right\rangle + x_1 \\
\text{subject to} \quad & \left\langle \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, X_1 \right\rangle + x_1 \quad = \quad 1.0\,, \\
& \left\langle \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, X_1 \right\rangle + x_0 + x_2 \quad = \quad 0.5\,, \\
& x_1 \geq \sqrt{x_0^2 + x_2^2}\,, \\
& X_1 \succeq \mathbf{0}\,.
\end{aligned}
\tag{13.7}
$$

The equality constraints are easily rewritten to the conic form, $(g_0, g_1) \in \{0\}^2$, by moving constants such that the right-hand-side becomes zero. The quadratic cone does not fit under the `VAR` keyword in this variable permutation. Instead, it takes a scalar constraint $(g_2, g_3, g_4) = (x_1, x_0, x_2) \in \mathcal{Q}^3$, with scalar variables constructed as $(x_0, x_1, x_2) \in \mathbb{R}^3$. Its formulation in the CBF format is reported in the following list

```
# File written using this version of the Conic Benchmark Format:
#      | Version 1.
VER
1

# The sense of the objective is:
#      | Minimize.
OBJSENSE
MIN

# One PSD variable of this size:
#      | Three times three.
PSDVAR
1
3

# Three scalar variables in this one conic domain:
#      | Three are free.
VAR
3 1
F 3

# Five scalar constraints with affine expressions in two conic domains:
#      | Two are fixed to zero.
#      | Three are in conic quadratic domain.
CON
5 2
L= 2
Q 3

# Five coordinates in F^{obj}_j coefficients:
#      | F^{obj}[0][0,0] = 2.0
#      | F^{obj}[0][1,0] = 1.0
#      | and more...
OBJFCOORD
5
0 0 0 2.0
0 1 0 1.0
0 1 1 2.0
```

```
0 2 1 1.0
0 2 2 2.0

# One coordinate in a^{obj}_j coefficients:
#     | a^{obj}[1] = 1.0
OBJACOORD
1
1 1.0

# Nine coordinates in F_ij coefficients:
#     | F[0,0][0,0] = 1.0
#     | F[0,0][1,1] = 1.0
#     | and more...
FCOORD
9
0 0 0 0 1.0
0 0 1 1 1.0
0 0 2 2 1.0
1 0 0 0 1.0
1 0 1 0 1.0
1 0 2 0 1.0
1 0 1 1 1.0
1 0 2 1 1.0
1 0 2 2 1.0

# Six coordinates in a_ij coefficients:
#     | a[0,1] = 1.0
#     | a[1,0] = 1.0
#     | and more...
ACOORD
6
0 1 1.0
1 0 1.0
1 2 1.0
2 1 1.0
3 0 1.0
4 2 1.0

# Two coordinates in b_i coefficients:
#     | b[0] = -1.0
#     | b[1] = -0.5
BCOORD
2
0 -1.0
1 -0.5
```

Mixing Semidefinite Variables and Linear Matrix Inequalities

The standard forms in semidefinite optimization are usually based either on semidefinite variables or linear matrix inequalities. In the CBF format, both forms are supported and can even be mixed as shown in.

$$
\begin{array}{rll}
\text{minimize} & \left\langle \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, X_1 \right\rangle + x_1 + x_2 + 1 \\
\text{subject to} & \left\langle \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, X_1 \right\rangle - x_1 - x_2 & \geq & 0.0, \\
& x_1 \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} + x_2 \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \succeq & \mathbf{0}, \\
& X_1 \succeq \mathbf{0}.
\end{array}
\tag{13.8}
$$

Its formulation in the CBF format is written in what follows

```
# File written using this version of the Conic Benchmark Format:
#       | Version 1.
VER
1

# The sense of the objective is:
#       | Minimize.
OBJSENSE
MIN

# One PSD variable of this size:
#       | Two times two.
PSDVAR
1
2

# Two scalar variables in this one conic domain:
#       | Two are free.
VAR
2 1
F 2

# One PSD constraint of this size:
#       | Two times two.
PSDCON
1
2

# One scalar constraint with an affine expression in this one conic domain:
#       | One is greater than or equal to zero.
CON
1 1
L+ 1

# Two coordinates in F^{obj}_j coefficients:
#       | F^{obj}[0][0,0] = 1.0
#       | F^{obj}[0][1,1] = 1.0
OBJFCOORD
2
0 0 0 1.0
0 1 1 1.0

# Two coordinates in a^{obj}_j coefficients:
#       | a^{obj}[0] = 1.0
#       | a^{obj}[1] = 1.0
OBJACOORD
2
0 1.0
1 1.0

# One coordinate in b^{obj} coefficient:
#       | b^{obj} = 1.0
OBJBCOORD
1.0

# One coordinate in F_ij coefficients:
#       | F[0,0][1,0] = 1.0
FCOORD
1
0 0 1 0 1.0

# Two coordinates in a_ij coefficients:
#       | a[0,0] = -1.0
```

```
#     | a[0,1] = -1.0
ACOORD
2
0 0 -1.0
0 1 -1.0

# Four coordinates in H_ij coefficients:
#     | H[0,0][1,0] = 1.0
#     | H[0,0][1,1] = 3.0
#     | and more...
HCOORD
4
0 0 1 0 1.0
0 0 1 1 3.0
0 1 0 0 3.0
0 1 1 0 1.0

# Two coordinates in D_i coefficients:
#     | D[0][0,0] = -1.0
#     | D[0][1,1] = -1.0
DCOORD
2
0 0 0 -1.0
0 1 1 -1.0
```

Optimization Over a Sequence of Objectives

The linear optimization problem (13.9), is defined for a sequence of objectives such that hotstarting from one to the next might be advantages.

$$
\begin{array}{rrcl}
\text{maximize}_k & g_k^{obj} \\
\text{subject to} & 50\, x_0 + 31 & \leq & 250\,, \\
& 3\, x_0 - 2x_1 & \geq & -4\,, \\
& x \in \mathbb{R}_+^2,
\end{array}
\tag{13.9}
$$

given,

1. $g_0^{obj} = x_0 + 0.64x_1$.

2. $g_1^{obj} = 1.11x_0 + 0.76x_1$.

3. $g_2^{obj} = 1.11x_0 + 0.85x_1$.

Its formulation in the CBF format is reported in Listing 13.5.

Listing 13.5: Problem (13.9) in CBF format.

```
# File written using this version of the Conic Benchmark Format:
#     | Version 1.
VER
1

# The sense of the objective is:
#     | Maximize.
OBJSENSE
MAX

# Two scalar variables in this one conic domain:
#     | Two are nonnegative.
VAR
2 1
L+ 2
```

```
# Two scalar constraints with affine expressions in these two conic domains:
#     | One is in the nonpositive domain.
#     | One is in the nonnegative domain.
CON
2 2
L- 1
L+ 1

# Two coordinates in a^{obj}_j coefficients:
#     | a^{obj}[0] = 1.0
#     | a^{obj}[1] = 0.64
OBJACOORD
2
0 1.0
1 0.64

# Four coordinates in a_ij coefficients:
#     | a[0,0] = 50.0
#     | a[1,0] = 3.0
#     | and more...
ACOORD
4
0 0 50.0
1 0 3.0
0 1 31.0
1 1 -2.0

# Two coordinates in b_i coefficients:
#     | b[0] = -250.0
#     | b[1] = 4.0
BCOORD
2
0 -250.0
1 4.0

# New problem instance defined in terms of changes.
CHANGE

# Two coordinate changes in a^{obj}_j coefficients. Now it is:
#     | a^{obj}[0] = 1.11
#     | a^{obj}[1] = 0.76
OBJACOORD
2
0 1.11
1 0.76

# New problem instance defined in terms of changes.
CHANGE

# One coordinate change in a^{obj}_j coefficients. Now it is:
#     | a^{obj}[0] = 1.11
#     | a^{obj}[1] = 0.85
OBJACOORD
1
1 0.85
```

13.5 The XML (OSiL) Format

MOSEK can write data in the standard OSiL xml format. For a definition of the OSiL format please see http://www.optimizationservices.org/.

Only linear constraints (possibly with integer variables) are supported. By default output files with the extension `.xml` are written in the OSiL format.

The parameter *MSK_IPAR_WRITE_XML_MODE* controls if the linear coefficients in the A matrix are written in row or column order.

13.6 The Task Format

The Task format is **MOSEK**'s native binary format. It contains a complete image of a **MOSEK** task, i.e.

- Problem data: Linear, conic quadratic, semidefinite and quadratic data
- Problem item names: Variable names, constraints names, cone names etc.
- Parameter settings
- Solutions

There are a few things to be aware of:

- The task format *does not* support General Convex problems since these are defined by arbitrary user-defined functions.
- Status of a solution read from a file will *always* be unknown.

The format is based on the *TAR* (USTar) file format. This means that the individual pieces of data in a `.task` file can be examined by unpacking it as a *TAR* file. Please note that the inverse may not work: Creating a file using *TAR* will most probably not create a valid **MOSEK** Task file since the order of the entries is important.

13.7 The JSON Format

MOSEK provides the possibility to read/write problems in valid JSON format.

JSON (JavaScript Object Notation) is a lightweight data-interchange format. It is easy for humans to read and write. It is easy for machines to parse and generate. It is based on a subset of the JavaScript Programming Language, Standard ECMA-262 3rd Edition - December 1999. JSON is a text format that is completely language independent but uses conventions that are familiar to programmers of the C-family of languages, including C, C++, C#, Java, JavaScript, Perl, Python, and many others. These properties make JSON an ideal data-interchange language.

The official JSON website http://www.json.org provides plenty of information along with the format definition.

MOSEK defines two JSON-like formats:

- *jtask*
- *jsol*

Warning: Despite being text-based human-readable formats, *jtask* and *jsol* files will include no indentation and no new-lines, in order to keep the files as compact as possible. We therefore strongly advise to use JSON viewer tools to inspect *jtask* and *jsol* files.

13.7.1 *jtask* format

It stores a problem instance. The *jtask* format contains the same information as a *task format*.

Even though a *jtask* file is human-readable, we do not recommend users to create it by hand, but to rely on **MOSEK**.

13.7.2 *jsol* format

It stores a problem solution. The *jsol* format contains all solutions and information items.

13.7.3 A *jtask* example

In Listing 13.6 we present a file in the *jtask* format that corresponds to the sample problem from `lo1.lp`. The listing has been formatted for readability.

Listing 13.6: A formatted *jtask* file for the `lo1.lp` example.

```
{
    "$schema":"http://mosek.com/json/schema#",
    "Task/INFO":{
        "taskname":"lo1",
        "numvar":4,
        "numcon":3,
        "numcone":0,
        "numbarvar":0,
        "numanz":9,
        "numsymmat":0,
        "mosekver":[
            8,
            0,
            0,
            9
        ]
    },
    "Task/data":{
        "var":{
            "name":[
                "x1",
                "x2",
                "x3",
                "x4"
            ],
            "bk":[
                "lo",
                "ra",
                "lo",
                "lo"
            ],
            "bl":[
                0.0,
                0.0,
                0.0,
                0.0
            ],
            "bu":[
                1e+30,
                1e+1,
                1e+30,
                1e+30
            ],
```

```json
                    "type":[
                        "cont",
                        "cont",
                        "cont",
                        "cont"
                    ]
                },
                "con":{
                    "name":[
                        "c1",
                        "c2",
                        "c3"
                    ],
                    "bk":[
                        "fx",
                        "lo",
                        "up"
                    ],
                    "bl":[
                        3e+1,
                        1.5e+1,
                            -1e+30
                    ],
                    "bu":[
                        3e+1,
                        1e+30,
                        2.5e+1
                    ]
                },
                "objective":{
                    "sense":"max",
                    "name":"obj",
                    "c":{
                        "subj":[
                            0,
                            1,
                            2,
                            3
                        ],
                        "val":[
                            3e+0,
                            1e+0,
                            5e+0,
                            1e+0
                        ]
                    },
                    "cfix":0.0
                },
                "A":{
                    "subi":[
                        0,
                        0,
                        0,
                        1,
                        1,
                        1,
                        1,
                        2,
                        2
                    ],
                    "subj":[
                        0,
                        1,
```

```
                2,
                0,
                1,
                2,
                3,
                1,
                3
            ],
            "val":[
                3e+0,
                1e+0,
                2e+0,
                2e+0,
                1e+0,
                3e+0,
                1e+0,
                2e+0,
                3e+0
            ]
        }
    },
    "Task/parameters":{
        "iparam":{
            "ANA_SOL_BASIS":"ON",
            "ANA_SOL_PRINT_VIOLATED":"OFF",
            "AUTO_SORT_A_BEFORE_OPT":"OFF",
            "AUTO_UPDATE_SOL_INFO":"OFF",
            "BASIS_SOLVE_USE_PLUS_ONE":"OFF",
            "BI_CLEAN_OPTIMIZER":"OPTIMIZER_FREE",
            "BI_IGNORE_MAX_ITER":"OFF",
            "BI_IGNORE_NUM_ERROR":"OFF",
            "BI_MAX_ITERATIONS":1000000,
            "CACHE_LICENSE":"ON",
            "CHECK_CONVEXITY":"CHECK_CONVEXITY_FULL",
            "COMPRESS_STATFILE":"ON",
            "CONCURRENT_NUM_OPTIMIZERS":2,
            "CONCURRENT_PRIORITY_DUAL_SIMPLEX":2,
            "CONCURRENT_PRIORITY_FREE_SIMPLEX":3,
            "CONCURRENT_PRIORITY_INTPNT":4,
            "CONCURRENT_PRIORITY_PRIMAL_SIMPLEX":1,
            "FEASREPAIR_OPTIMIZE":"FEASREPAIR_OPTIMIZE_NONE",
            "INFEAS_GENERIC_NAMES":"OFF",
            "INFEAS_PREFER_PRIMAL":"ON",
            "INFEAS_REPORT_AUTO":"OFF",
            "INFEAS_REPORT_LEVEL":1,
            "INTPNT_BASIS":"BI_ALWAYS",
            "INTPNT_DIFF_STEP":"ON",
            "INTPNT_FACTOR_DEBUG_LVL":0,
            "INTPNT_FACTOR_METHOD":0,
            "INTPNT_HOTSTART":"INTPNT_HOTSTART_NONE",
            "INTPNT_MAX_ITERATIONS":400,
            "INTPNT_MAX_NUM_COR":-1,
            "INTPNT_MAX_NUM_REFINEMENT_STEPS":-1,
            "INTPNT_OFF_COL_TRH":40,
            "INTPNT_ORDER_METHOD":"ORDER_METHOD_FREE",
            "INTPNT_REGULARIZATION_USE":"ON",
            "INTPNT_SCALING":"SCALING_FREE",
            "INTPNT_SOLVE_FORM":"SOLVE_FREE",
            "INTPNT_STARTING_POINT":"STARTING_POINT_FREE",
            "LIC_TRH_EXPIRY_WRN":7,
            "LICENSE_DEBUG":"OFF",
            "LICENSE_PAUSE_TIME":0,
            "LICENSE_SUPPRESS_EXPIRE_WRNS":"OFF",
```

```json
            "LICENSE_WAIT":"OFF",
            "LOG":10,
            "LOG_ANA_PRO":1,
            "LOG_BI":4,
            "LOG_BI_FREQ":2500,
            "LOG_CHECK_CONVEXITY":0,
            "LOG_CONCURRENT":1,
            "LOG_CUT_SECOND_OPT":1,
            "LOG_EXPAND":0,
            "LOG_FACTOR":1,
            "LOG_FEAS_REPAIR":1,
            "LOG_FILE":1,
            "LOG_HEAD":1,
            "LOG_INFEAS_ANA":1,
            "LOG_INTPNT":4,
            "LOG_MIO":4,
            "LOG_MIO_FREQ":1000,
            "LOG_OPTIMIZER":1,
            "LOG_ORDER":1,
            "LOG_PRESOLVE":1,
            "LOG_RESPONSE":0,
            "LOG_SENSITIVITY":1,
            "LOG_SENSITIVITY_OPT":0,
            "LOG_SIM":4,
            "LOG_SIM_FREQ":1000,
            "LOG_SIM_MINOR":1,
            "LOG_STORAGE":1,
            "MAX_NUM_WARNINGS":10,
            "MIO_BRANCH_DIR":"BRANCH_DIR_FREE",
            "MIO_CONSTRUCT_SOL":"OFF",
            "MIO_CUT_CLIQUE":"ON",
            "MIO_CUT_CMIR":"ON",
            "MIO_CUT_GMI":"ON",
            "MIO_CUT_KNAPSACK_COVER":"OFF",
            "MIO_HEURISTIC_LEVEL":-1,
            "MIO_MAX_NUM_BRANCHES":-1,
            "MIO_MAX_NUM_RELAXS":-1,
            "MIO_MAX_NUM_SOLUTIONS":-1,
            "MIO_MODE":"MIO_MODE_SATISFIED",
            "MIO_MT_USER_CB":"ON",
            "MIO_NODE_OPTIMIZER":"OPTIMIZER_FREE",
            "MIO_NODE_SELECTION":"MIO_NODE_SELECTION_FREE",
            "MIO_PERSPECTIVE_REFORMULATE":"ON",
            "MIO_PROBING_LEVEL":-1,
            "MIO_RINS_MAX_NODES":-1,
            "MIO_ROOT_OPTIMIZER":"OPTIMIZER_FREE",
            "MIO_ROOT_REPEAT_PRESOLVE_LEVEL":-1,
            "MT_SPINCOUNT":0,
            "NUM_THREADS":0,
            "OPF_MAX_TERMS_PER_LINE":5,
            "OPF_WRITE_HEADER":"ON",
            "OPF_WRITE_HINTS":"ON",
            "OPF_WRITE_PARAMETERS":"OFF",
            "OPF_WRITE_PROBLEM":"ON",
            "OPF_WRITE_SOL_BAS":"ON",
            "OPF_WRITE_SOL_ITG":"ON",
            "OPF_WRITE_SOL_ITR":"ON",
            "OPF_WRITE_SOLUTIONS":"OFF",
            "OPTIMIZER":"OPTIMIZER_FREE",
            "PARAM_READ_CASE_NAME":"ON",
            "PARAM_READ_IGN_ERROR":"OFF",
            "PRESOLVE_ELIMINATOR_MAX_FILL":-1,
            "PRESOLVE_ELIMINATOR_MAX_NUM_TRIES":-1,
```

```
            "PRESOLVE_LEVEL":-1,
            "PRESOLVE_LINDEP_ABS_WORK_TRH":100,
            "PRESOLVE_LINDEP_REL_WORK_TRH":100,
            "PRESOLVE_LINDEP_USE":"ON",
            "PRESOLVE_MAX_NUM_REDUCTIONS":-1,
            "PRESOLVE_USE":"PRESOLVE_MODE_FREE",
            "PRIMAL_REPAIR_OPTIMIZER":"OPTIMIZER_FREE",
            "QO_SEPARABLE_REFORMULATION":"OFF",
            "READ_DATA_COMPRESSED":"COMPRESS_FREE",
            "READ_DATA_FORMAT":"DATA_FORMAT_EXTENSION",
            "READ_DEBUG":"OFF",
            "READ_KEEP_FREE_CON":"OFF",
            "READ_LP_DROP_NEW_VARS_IN_BOU":"OFF",
            "READ_LP_QUOTED_NAMES":"ON",
            "READ_MPS_FORMAT":"MPS_FORMAT_FREE",
            "READ_MPS_WIDTH":1024,
            "READ_TASK_IGNORE_PARAM":"OFF",
            "SENSITIVITY_ALL":"OFF",
            "SENSITIVITY_OPTIMIZER":"OPTIMIZER_FREE_SIMPLEX",
            "SENSITIVITY_TYPE":"SENSITIVITY_TYPE_BASIS",
            "SIM_BASIS_FACTOR_USE":"ON",
            "SIM_DEGEN":"SIM_DEGEN_FREE",
            "SIM_DUAL_CRASH":90,
            "SIM_DUAL_PHASEONE_METHOD":0,
            "SIM_DUAL_RESTRICT_SELECTION":50,
            "SIM_DUAL_SELECTION":"SIM_SELECTION_FREE",
            "SIM_EXPLOIT_DUPVEC":"SIM_EXPLOIT_DUPVEC_OFF",
            "SIM_HOTSTART":"SIM_HOTSTART_FREE",
            "SIM_HOTSTART_LU":"ON",
            "SIM_INTEGER":0,
            "SIM_MAX_ITERATIONS":10000000,
            "SIM_MAX_NUM_SETBACKS":250,
            "SIM_NON_SINGULAR":"ON",
            "SIM_PRIMAL_CRASH":90,
            "SIM_PRIMAL_PHASEONE_METHOD":0,
            "SIM_PRIMAL_RESTRICT_SELECTION":50,
            "SIM_PRIMAL_SELECTION":"SIM_SELECTION_FREE",
            "SIM_REFACTOR_FREQ":0,
            "SIM_REFORMULATION":"SIM_REFORMULATION_OFF",
            "SIM_SAVE_LU":"OFF",
            "SIM_SCALING":"SCALING_FREE",
            "SIM_SCALING_METHOD":"SCALING_METHOD_POW2",
            "SIM_SOLVE_FORM":"SOLVE_FREE",
            "SIM_STABILITY_PRIORITY":50,
            "SIM_SWITCH_OPTIMIZER":"OFF",
            "SOL_FILTER_KEEP_BASIC":"OFF",
            "SOL_FILTER_KEEP_RANGED":"OFF",
            "SOL_READ_NAME_WIDTH":-1,
            "SOL_READ_WIDTH":1024,
            "SOLUTION_CALLBACK":"OFF",
            "TIMING_LEVEL":1,
            "WRITE_BAS_CONSTRAINTS":"ON",
            "WRITE_BAS_HEAD":"ON",
            "WRITE_BAS_VARIABLES":"ON",
            "WRITE_DATA_COMPRESSED":0,
            "WRITE_DATA_FORMAT":"DATA_FORMAT_EXTENSION",
            "WRITE_DATA_PARAM":"OFF",
            "WRITE_FREE_CON":"OFF",
            "WRITE_GENERIC_NAMES":"OFF",
            "WRITE_GENERIC_NAMES_IO":1,
            "WRITE_IGNORE_INCOMPATIBLE_CONIC_ITEMS":"OFF",
            "WRITE_IGNORE_INCOMPATIBLE_ITEMS":"OFF",
            "WRITE_IGNORE_INCOMPATIBLE_NL_ITEMS":"OFF",
```

```
                    "WRITE_IGNORE_INCOMPATIBLE_PSD_ITEMS":"OFF",
                    "WRITE_INT_CONSTRAINTS":"ON",
                    "WRITE_INT_HEAD":"ON",
                    "WRITE_INT_VARIABLES":"ON",
                    "WRITE_LP_FULL_OBJ":"ON",
                    "WRITE_LP_LINE_WIDTH":80,
                    "WRITE_LP_QUOTED_NAMES":"ON",
                    "WRITE_LP_STRICT_FORMAT":"OFF",
                    "WRITE_LP_TERMS_PER_LINE":10,
                    "WRITE_MPS_FORMAT":"MPS_FORMAT_FREE",
                    "WRITE_MPS_INT":"ON",
                    "WRITE_PRECISION":15,
                    "WRITE_SOL_BARVARIABLES":"ON",
                    "WRITE_SOL_CONSTRAINTS":"ON",
                    "WRITE_SOL_HEAD":"ON",
                    "WRITE_SOL_IGNORE_INVALID_NAMES":"OFF",
                    "WRITE_SOL_VARIABLES":"ON",
                    "WRITE_TASK_INC_SOL":"ON",
                    "WRITE_XML_MODE":"WRITE_XML_MODE_ROW"
            },
            "dparam":{
                    "ANA_SOL_INFEAS_TOL":1e-6,
                    "BASIS_REL_TOL_S":1e-12,
                    "BASIS_TOL_S":1e-6,
                    "BASIS_TOL_X":1e-6,
                    "CHECK_CONVEXITY_REL_TOL":1e-10,
                    "DATA_TOL_AIJ":1e-12,
                    "DATA_TOL_AIJ_HUGE":1e+20,
                    "DATA_TOL_AIJ_LARGE":1e+10,
                    "DATA_TOL_BOUND_INF":1e+16,
                    "DATA_TOL_BOUND_WRN":1e+8,
                    "DATA_TOL_C_HUGE":1e+16,
                    "DATA_TOL_CJ_LARGE":1e+8,
                    "DATA_TOL_QIJ":1e-16,
                    "DATA_TOL_X":1e-8,
                    "FEASREPAIR_TOL":1e-10,
                    "INTPNT_CO_TOL_DFEAS":1e-8,
                    "INTPNT_CO_TOL_INFEAS":1e-10,
                    "INTPNT_CO_TOL_MU_RED":1e-8,
                    "INTPNT_CO_TOL_NEAR_REL":1e+3,
                    "INTPNT_CO_TOL_PFEAS":1e-8,
                    "INTPNT_CO_TOL_REL_GAP":1e-7,
                    "INTPNT_NL_MERIT_BAL":1e-4,
                    "INTPNT_NL_TOL_DFEAS":1e-8,
                    "INTPNT_NL_TOL_MU_RED":1e-12,
                    "INTPNT_NL_TOL_NEAR_REL":1e+3,
                    "INTPNT_NL_TOL_PFEAS":1e-8,
                    "INTPNT_NL_TOL_REL_GAP":1e-6,
                    "INTPNT_NL_TOL_REL_STEP":9.95e-1,
                    "INTPNT_QO_TOL_DFEAS":1e-8,
                    "INTPNT_QO_TOL_INFEAS":1e-10,
                    "INTPNT_QO_TOL_MU_RED":1e-8,
                    "INTPNT_QO_TOL_NEAR_REL":1e+3,
                    "INTPNT_QO_TOL_PFEAS":1e-8,
                    "INTPNT_QO_TOL_REL_GAP":1e-8,
                    "INTPNT_TOL_DFEAS":1e-8,
                    "INTPNT_TOL_DSAFE":1e+0,
                    "INTPNT_TOL_INFEAS":1e-10,
                    "INTPNT_TOL_MU_RED":1e-16,
                    "INTPNT_TOL_PATH":1e-8,
                    "INTPNT_TOL_PFEAS":1e-8,
                    "INTPNT_TOL_PSAFE":1e+0,
                    "INTPNT_TOL_REL_GAP":1e-8,
```

```
            "INTPNT_TOL_REL_STEP":9.999e-1,
            "INTPNT_TOL_STEP_SIZE":1e-6,
            "LOWER_OBJ_CUT":-1e+30,
            "LOWER_OBJ_CUT_FINITE_TRH":-5e+29,
            "MIO_DISABLE_TERM_TIME":-1e+0,
            "MIO_MAX_TIME":-1e+0,
            "MIO_MAX_TIME_APRX_OPT":6e+1,
            "MIO_NEAR_TOL_ABS_GAP":0.0,
            "MIO_NEAR_TOL_REL_GAP":1e-3,
            "MIO_REL_GAP_CONST":1e-10,
            "MIO_TOL_ABS_GAP":0.0,
            "MIO_TOL_ABS_RELAX_INT":1e-5,
            "MIO_TOL_FEAS":1e-6,
            "MIO_TOL_REL_DUAL_BOUND_IMPROVEMENT":0.0,
            "MIO_TOL_REL_GAP":1e-4,
            "MIO_TOL_X":1e-6,
            "OPTIMIZER_MAX_TIME":-1e+0,
            "PRESOLVE_TOL_ABS_LINDEP":1e-6,
            "PRESOLVE_TOL_AIJ":1e-12,
            "PRESOLVE_TOL_REL_LINDEP":1e-10,
            "PRESOLVE_TOL_S":1e-8,
            "PRESOLVE_TOL_X":1e-8,
            "QCQO_REFORMULATE_REL_DROP_TOL":1e-15,
            "SEMIDEFINITE_TOL_APPROX":1e-10,
            "SIM_LU_TOL_REL_PIV":1e-2,
            "SIMPLEX_ABS_TOL_PIV":1e-7,
            "UPPER_OBJ_CUT":1e+30,
            "UPPER_OBJ_CUT_FINITE_TRH":5e+29
        },
        "sparam":{
            "BAS_SOL_FILE_NAME":"",
            "DATA_FILE_NAME":"examples/tools/data/lo1.mps",
            "DEBUG_FILE_NAME":"",
            "INT_SOL_FILE_NAME":"",
            "ITR_SOL_FILE_NAME":"",
            "MIO_DEBUG_STRING":"",
            "PARAM_COMMENT_SIGN":"%%",
            "PARAM_READ_FILE_NAME":"",
            "PARAM_WRITE_FILE_NAME":"",
            "READ_MPS_BOU_NAME":"",
            "READ_MPS_OBJ_NAME":"",
            "READ_MPS_RAN_NAME":"",
            "READ_MPS_RHS_NAME":"",
            "SENSITIVITY_FILE_NAME":"",
            "SENSITIVITY_RES_FILE_NAME":"",
            "SOL_FILTER_XC_LOW":"",
            "SOL_FILTER_XC_UPR":"",
            "SOL_FILTER_XX_LOW":"",
            "SOL_FILTER_XX_UPR":"",
            "STAT_FILE_NAME":"",
            "STAT_KEY":"",
            "STAT_NAME":"",
            "WRITE_LP_GEN_VAR_NAME":"XMSKGEN"
        }
    }
}
```

13.8 The Solution File Format

MOSEK provides several solution files depending on the problem type and the optimizer used:

- *basis solution file* (extension `.bas`) if the problem is optimized using the simplex optimizer or basis identification is performed,

- *interior solution file* (extension `.sol`) if a problem is optimized using the interior-point optimizer and no basis identification is required,

- *integer solution file* (extension `.int`) if the problem contains integer constrained variables.

All solution files have the format:

```
NAME                : <problem name>
PROBLEM STATUS      : <status of the problem>
SOLUTION STATUS     : <status of the solution>
OBJECTIVE NAME      : <name of the objective function>
PRIMAL OBJECTIVE    : <primal objective value corresponding to the solution>
DUAL OBJECTIVE      : <dual objective value corresponding to the solution>
CONSTRAINTS
INDEX  NAME    AT ACTIVITY    LOWER LIMIT    UPPER LIMIT    DUAL LOWER    DUAL UPPER
?      <name>  ?? <a value>   <a value>      <a value>      <a value>     <a value>
VARIABLES
INDEX  NAME    AT ACTIVITY    LOWER LIMIT    UPPER LIMIT    DUAL LOWER    DUAL UPPER    CONIC␣
↪DUAL
?      <name>  ?? <a value>   <a value>      <a value>      <a value>     <a value>     <a value>
```

In the example the fields `?` and `<>` will be filled with problem and solution specific information. As can be observed a solution report consists of three sections, i.e.

- `HEADER` In this section, first the name of the problem is listed and afterwards the problem and solution status are shown. Next the primal and dual objective values are displayed.

- `CONSTRAINTS` For each constraint i of the form

$$l_i^c \leq \sum_{j=1}^{n} a_{ij} x_j \leq u_i^c, \tag{13.10}$$

the following information is listed:

- `INDEX`: A sequential index assigned to the constraint by **MOSEK**

- `NAME`: The name of the constraint assigned by the user.

- `AT`: The status of the constraint. In Table 13.4 the possible values of the status keys and their interpretation are shown.

Table 13.4: Status keys.

Status key	Interpretation
UN	Unknown status
BS	Is basic
SB	Is superbasic
LL	Is at the lower limit (bound)
UL	Is at the upper limit (bound)
EQ	Lower limit is identical to upper limit
**	Is infeasible i.e. the lower limit is greater than the upper limit.

- `ACTIVITY`: the quantity $\sum_{j=1}^{n} a_{ij} x_j^*$, where x^* is the value of the primal solution.

- `LOWER LIMIT`: the quantity l_i^c (see (13.10).)

- `UPPER LIMIT`: the quantity u_i^c (see (13.10).)

- `DUAL LOWER`: the dual multiplier corresponding to the lower limit on the constraint.

- `DUAL UPPER`: the dual multiplier corresponding to the upper limit on the constraint.

- **VARIABLES** The last section of the solution report lists information about the variables. This information has a similar interpretation as for the constraints. However, the column with the header `CONIC DUAL` is included for problems having one or more conic constraints. This column shows the dual variables corresponding to the conic constraints.

Example: `lo1.sol`

In Listing 13.7 we show the solution file for the `lo1.opf` problem.

Listing 13.7: An example of `.sol` file.

```
NAME                   :
PROBLEM STATUS        : PRIMAL_AND_DUAL_FEASIBLE
SOLUTION STATUS       : OPTIMAL
OBJECTIVE NAME        : obj
PRIMAL OBJECTIVE      : 8.33333333e+01
DUAL OBJECTIVE        : 8.33333332e+01

CONSTRAINTS
INDEX      NAME         AT ACTIVITY              LOWER LIMIT        UPPER LIMIT        ␣
↪DUAL LOWER            DUAL UPPER
0         c1           EQ 3.00000000000000e+01  3.00000000e+01     3.00000000e+01     -0.
↪00000000000000e+00   -2.49999999741654e+00
1         c2           SB 5.33333333049188e+01  1.50000000e+01     NONE               2.
↪09157603759397e-10   -0.00000000000000e+00
2         c3           UL 2.49999999842049e+01  NONE               2.50000000e+01     -0.
↪00000000000000e+00   -3.33333332895110e-01

VARIABLES
INDEX      NAME         AT ACTIVITY              LOWER LIMIT        UPPER LIMIT        ␣
↪DUAL LOWER            DUAL UPPER
0         x1           LL 1.67020427073508e-09  0.00000000e+00     NONE               -4.
↪49999999528055e+00   -0.00000000000000e+00
1         x2           LL 2.93510446280504e-09  0.00000000e+00     1.00000000e+01     -2.
↪16666666494916e+00   6.20863861687316e-10
2         x3           SB 1.49999999899425e+01  0.00000000e+00     NONE               -8.
↪79123177454657e-10   -0.00000000000000e+00
3         x4           SB 8.33333332273116e+00  0.00000000e+00     NONE               -1.
↪69795978899185e-09   -0.00000000000000e+00
```

LIST OF EXAMPLES

List of examples shipped in the distribution of Rmosek Package:

Table 14.1: List of distributed examples

File	Description
concurrent1.R	Shows how to use the concurrent optimizer
cqo1.R	A simple conic quadratic problem
cqo1_preallocatedcones.R	A simple conic quadratic problem with preallocated cones
lo1.R	A simple linear problem
milo1.R	A simple mixed-integer linear problem
parameters.R	Shows how to set optimizer parameters
qo1.R	A simple quadratic problem
scopt1.R	Shows how to solve a simple non-linear separable problem using the SCopt interface
sdo1.R	A simple semidefinite optimization problem
simple.R	A simple I/O example: read problem from a file, solve and write solutions
solutionquality.R	Demonstrates how to examine the quality of a solution

Additional examples can be found on the **MOSEK** website and in other **MOSEK** publications.

INTERFACE CHANGES

The section show interface-specific changes to the **MOSEK** Rmosek Package in version 8. See the release notes for general changes and new features of the **MOSEK** Optimization Suite.

15.1 Parameters

Added

- *MSK_DPAR_DATA_SYM_MAT_TOL*
- *MSK_DPAR_DATA_SYM_MAT_TOL_HUGE*
- *MSK_DPAR_DATA_SYM_MAT_TOL_LARGE*
- *MSK_DPAR_INTPNT_QO_TOL_DFEAS*
- *MSK_DPAR_INTPNT_QO_TOL_INFEAS*
- *MSK_DPAR_INTPNT_QO_TOL_MU_RED*
- *MSK_DPAR_INTPNT_QO_TOL_NEAR_REL*
- *MSK_DPAR_INTPNT_QO_TOL_PFEAS*
- *MSK_DPAR_INTPNT_QO_TOL_REL_GAP*
- *MSK_DPAR_SEMIDEFINITE_TOL_APPROX*
- *MSK_IPAR_INTPNT_MULTI_THREAD*
- *MSK_IPAR_LICENSE_TRH_EXPIRY_WRN*
- *MSK_IPAR_LOG_ANA_PRO*
- *MSK_IPAR_MIO_CUT_CLIQUE*
- *MSK_IPAR_MIO_CUT_GMI*
- *MSK_IPAR_MIO_CUT_IMPLIED_BOUND*
- *MSK_IPAR_MIO_CUT_KNAPSACK_COVER*
- *MSK_IPAR_MIO_CUT_SELECTION_LEVEL*
- *MSK_IPAR_MIO_PERSPECTIVE_REFORMULATE*
- *MSK_IPAR_MIO_ROOT_REPEAT_PRESOLVE_LEVEL*
- *MSK_IPAR_MIO_VB_DETECTION_LEVEL*
- *MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL*
- *MSK_IPAR_REMOVE_UNUSED_SOLUTIONS*
- *MSK_IPAR_WRITE_LP_FULL_OBJ*

- *MSK_IPAR_WRITE_MPS_FORMAT*
- *MSK_SPAR_REMOTE_ACCESS_TOKEN*

Removed

- MSK_DPAR_FEASREPAIR_TOL
- MSK_DPAR_MIO_HEURISTIC_TIME
- MSK_DPAR_MIO_MAX_TIME_APRX_OPT
- MSK_DPAR_MIO_REL_ADD_CUT_LIMITED
- MSK_DPAR_MIO_TOL_MAX_CUT_FRAC_RHS
- MSK_DPAR_MIO_TOL_MIN_CUT_FRAC_RHS
- MSK_DPAR_MIO_TOL_REL_RELAX_INT
- MSK_DPAR_MIO_TOL_X
- MSK_DPAR_NONCONVEX_TOL_FEAS
- MSK_DPAR_NONCONVEX_TOL_OPT
- MSK_IPAR_ALLOC_ADD_QNZ
- MSK_IPAR_CONCURRENT_NUM_OPTIMIZERS
- MSK_IPAR_CONCURRENT_PRIORITY_DUAL_SIMPLEX
- MSK_IPAR_CONCURRENT_PRIORITY_FREE_SIMPLEX
- MSK_IPAR_CONCURRENT_PRIORITY_INTPNT
- MSK_IPAR_CONCURRENT_PRIORITY_PRIMAL_SIMPLEX
- MSK_IPAR_FEASREPAIR_OPTIMIZE
- MSK_IPAR_INTPNT_FACTOR_DEBUG_LVL
- MSK_IPAR_INTPNT_FACTOR_METHOD
- MSK_IPAR_LIC_TRH_EXPIRY_WRN
- MSK_IPAR_LOG_CONCURRENT
- MSK_IPAR_LOG_FACTOR
- MSK_IPAR_LOG_HEAD
- MSK_IPAR_LOG_NONCONVEX
- MSK_IPAR_LOG_OPTIMIZER
- MSK_IPAR_LOG_PARAM
- MSK_IPAR_LOG_SIM_NETWORK_FREQ
- MSK_IPAR_MIO_BRANCH_PRIORITIES_USE
- MSK_IPAR_MIO_CONT_SOL
- MSK_IPAR_MIO_CUT_CG
- MSK_IPAR_MIO_CUT_LEVEL_ROOT
- MSK_IPAR_MIO_CUT_LEVEL_TREE
- MSK_IPAR_MIO_FEASPUMP_LEVEL
- MSK_IPAR_MIO_HOTSTART
- MSK_IPAR_MIO_KEEP_BASIS

- MSK_IPAR_MIO_LOCAL_BRANCH_NUMBER

- MSK_IPAR_MIO_OPTIMIZER_MODE

- MSK_IPAR_MIO_PRESOLVE_AGGREGATE

- MSK_IPAR_MIO_PRESOLVE_PROBING

- MSK_IPAR_MIO_PRESOLVE_USE

- MSK_IPAR_MIO_STRONG_BRANCH

- MSK_IPAR_MIO_USE_MULTITHREADED_OPTIMIZER

- MSK_IPAR_NONCONVEX_MAX_ITERATIONS

- MSK_IPAR_PRESOLVE_ELIM_FILL

- MSK_IPAR_PRESOLVE_ELIMINATOR_USE

- MSK_IPAR_QO_SEPARABLE_REFORMULATION

- MSK_IPAR_READ_ANZ

- MSK_IPAR_READ_CON

- MSK_IPAR_READ_CONE

- MSK_IPAR_READ_MPS_KEEP_INT

- MSK_IPAR_READ_MPS_OBJ_SENSE

- MSK_IPAR_READ_MPS_RELAX

- MSK_IPAR_READ_QNZ

- MSK_IPAR_READ_VAR

- MSK_IPAR_SIM_INTEGER

- MSK_IPAR_WARNING_LEVEL

- MSK_IPAR_WRITE_IGNORE_INCOMPATIBLE_CONIC_ITEMS

- MSK_IPAR_WRITE_IGNORE_INCOMPATIBLE_NL_ITEMS

- MSK_IPAR_WRITE_IGNORE_INCOMPATIBLE_PSD_ITEMS

- MSK_SPAR_FEASREPAIR_NAME_PREFIX

- MSK_SPAR_FEASREPAIR_NAME_SEPARATOR

- MSK_SPAR_FEASREPAIR_NAME_WSUMVIOL

15.2 Constants

Added

- *"MSK_BRANCH_DIR_FAR"*

- *"MSK_BRANCH_DIR_GUIDED"*

- *"MSK_BRANCH_DIR_NEAR"*

- *"MSK_BRANCH_DIR_PSEUDOCOST"*

- *"MSK_BRANCH_DIR_ROOT_LP"*

- *"MSK_CALLBACK_BEGIN_ROOT_CUTGEN"*

- *"MSK_CALLBACK_BEGIN_TO_CONIC"*

- *"MSK_CALLBACK_END_ROOT_CUTGEN"*
- *"MSK_CALLBACK_END_TO_CONIC"*
- *"MSK_CALLBACK_IM_ROOT_CUTGEN"*
- *"MSK_CALLBACK_SOLVING_REMOTE"*
- *"MSK_DATA_FORMAT_JSON_TASK"*
- *"MSK_DINF_MIO_CLIQUE_SEPARATION_TIME"*
- *"MSK_DINF_MIO_CMIR_SEPARATION_TIME"*
- *"MSK_DINF_MIO_GMI_SEPARATION_TIME"*
- *"MSK_DINF_MIO_IMPLIED_BOUND_TIME"*
- *"MSK_DINF_MIO_KNAPSACK_COVER_SEPARATION_TIME"*
- *"MSK_DINF_QCQO_REFORMULATE_MAX_PERTURBATION"*
- *"MSK_DINF_QCQO_REFORMULATE_WORST_CHOLESKY_COLUMN_SCALING"*
- *"MSK_DINF_QCQO_REFORMULATE_WORST_CHOLESKY_DIAG_SCALING"*
- *"MSK_DINF_SOL_BAS_NRM_BARX"*
- *"MSK_DINF_SOL_BAS_NRM_SLC"*
- *"MSK_DINF_SOL_BAS_NRM_SLX"*
- *"MSK_DINF_SOL_BAS_NRM_SUC"*
- *"MSK_DINF_SOL_BAS_NRM_SUX"*
- *"MSK_DINF_SOL_BAS_NRM_XC"*
- *"MSK_DINF_SOL_BAS_NRM_XX"*
- *"MSK_DINF_SOL_BAS_NRM_Y"*
- *"MSK_DINF_SOL_ITG_NRM_BARX"*
- *"MSK_DINF_SOL_ITG_NRM_XC"*
- *"MSK_DINF_SOL_ITG_NRM_XX"*
- *"MSK_DINF_SOL_ITR_NRM_BARS"*
- *"MSK_DINF_SOL_ITR_NRM_BARX"*
- *"MSK_DINF_SOL_ITR_NRM_SLC"*
- *"MSK_DINF_SOL_ITR_NRM_SLX"*
- *"MSK_DINF_SOL_ITR_NRM_SNX"*
- *"MSK_DINF_SOL_ITR_NRM_SUC"*
- *"MSK_DINF_SOL_ITR_NRM_SUX"*
- *"MSK_DINF_SOL_ITR_NRM_XC"*
- *"MSK_DINF_SOL_ITR_NRM_XX"*
- *"MSK_DINF_SOL_ITR_NRM_Y"*
- *"MSK_DINF_TO_CONIC_TIME"*
- *"MSK_IINF_MIO_ABSGAP_SATISFIED"*
- *"MSK_IINF_MIO_CLIQUE_TABLE_SIZE"*
- *"MSK_IINF_MIO_NEAR_ABSGAP_SATISFIED"*
- *"MSK_IINF_MIO_NEAR_RELGAP_SATISFIED"*

- *"MSK_IINF_MIO_NODE_DEPTH"*
- *"MSK_IINF_MIO_NUM_CMIR_CUTS"*
- *"MSK_IINF_MIO_NUM_IMPLIED_BOUND_CUTS"*
- *"MSK_IINF_MIO_NUM_KNAPSACK_COVER_CUTS"*
- *"MSK_IINF_MIO_NUM_REPEATED_PRESOLVE"*
- *"MSK_IINF_MIO_PRESOLVED_NUMBIN"*
- *"MSK_IINF_MIO_PRESOLVED_NUMCON"*
- *"MSK_IINF_MIO_PRESOLVED_NUMCONT"*
- *"MSK_IINF_MIO_PRESOLVED_NUMINT"*
- *"MSK_IINF_MIO_PRESOLVED_NUMVAR"*
- *"MSK_IINF_MIO_RELGAP_SATISFIED"*
- *"MSK_LIINF_MIO_PRESOLVED_ANZ"*
- *"MSK_LIINF_MIO_SIM_MAXITER_SETBACKS"*
- *"MSK_MPS_FORMAT_CPLEX"*
- *"MSK_SOL_STA_DUAL_ILLPOSED_CER"*
- *"MSK_SOL_STA_PRIM_ILLPOSED_CER"*

Changed

- *"MSK_SOL_STA_INTEGER_OPTIMAL"*
- *"MSK_SOL_STA_NEAR_DUAL_FEAS"*
- *"MSK_SOL_STA_NEAR_DUAL_INFEAS_CER"*
- *"MSK_SOL_STA_NEAR_INTEGER_OPTIMAL"*
- *"MSK_SOL_STA_NEAR_OPTIMAL"*
- *"MSK_SOL_STA_NEAR_PRIM_AND_DUAL_FEAS"*
- *"MSK_SOL_STA_NEAR_PRIM_FEAS"*
- *"MSK_SOL_STA_NEAR_PRIM_INFEAS_CER"*
- *"MSK_LICENSE_BUFFER_LENGTH"*

Removed

- MSK_CALLBACKCODE_BEGIN_CONCURRENT
- MSK_CALLBACKCODE_BEGIN_NETWORK_DUAL_SIMPLEX
- MSK_CALLBACKCODE_BEGIN_NETWORK_PRIMAL_SIMPLEX
- MSK_CALLBACKCODE_BEGIN_NETWORK_SIMPLEX
- MSK_CALLBACKCODE_BEGIN_NONCONVEX
- MSK_CALLBACKCODE_BEGIN_PRIMAL_DUAL_SIMPLEX
- MSK_CALLBACKCODE_BEGIN_PRIMAL_DUAL_SIMPLEX_BI
- MSK_CALLBACKCODE_BEGIN_SIMPLEX_NETWORK_DETECT
- MSK_CALLBACKCODE_END_CONCURRENT

- MSK_CALLBACKCODE_END_NETWORK_DUAL_SIMPLEX

- MSK_CALLBACKCODE_END_NETWORK_PRIMAL_SIMPLEX

- MSK_CALLBACKCODE_END_NETWORK_SIMPLEX

- MSK_CALLBACKCODE_END_NONCONVEX

- MSK_CALLBACKCODE_END_PRIMAL_DUAL_SIMPLEX

- MSK_CALLBACKCODE_END_PRIMAL_DUAL_SIMPLEX_BI

- MSK_CALLBACKCODE_END_SIMPLEX_NETWORK_DETECT

- MSK_CALLBACKCODE_IM_MIO_PRESOLVE

- MSK_CALLBACKCODE_IM_NETWORK_DUAL_SIMPLEX

- MSK_CALLBACKCODE_IM_NETWORK_PRIMAL_SIMPLEX

- MSK_CALLBACKCODE_IM_NONCONVEX

- MSK_CALLBACKCODE_IM_PRIMAL_DUAL_SIMPLEX

- MSK_CALLBACKCODE_NONCOVEX

- MSK_CALLBACKCODE_UPDATE_NETWORK_DUAL_SIMPLEX

- MSK_CALLBACKCODE_UPDATE_NETWORK_PRIMAL_SIMPLEX

- MSK_CALLBACKCODE_UPDATE_NONCONVEX

- MSK_CALLBACKCODE_UPDATE_PRIMAL_DUAL_SIMPLEX

- MSK_CALLBACKCODE_UPDATE_PRIMAL_DUAL_SIMPLEX_BI

- MSK_DINFITEM_BI_CLEAN_PRIMAL_DUAL_TIME

- MSK_DINFITEM_CONCURRENT_TIME

- MSK_DINFITEM_MIO_CG_SEPERATION_TIME

- MSK_DINFITEM_MIO_CMIR_SEPERATION_TIME

- MSK_DINFITEM_SIM_NETWORK_DUAL_TIME

- MSK_DINFITEM_SIM_NETWORK_PRIMAL_TIME

- MSK_DINFITEM_SIM_NETWORK_TIME

- MSK_DINFITEM_SIM_PRIMAL_DUAL_TIME

- MSK_FEATURE_PTOM

- MSK_FEATURE_PTOX

- MSK_IINFITEM_CONCURRENT_FASTEST_OPTIMIZER

- MSK_IINFITEM_MIO_NUM_BASIS_CUTS

- MSK_IINFITEM_MIO_NUM_CARDGUB_CUTS

- MSK_IINFITEM_MIO_NUM_COEF_REDC_CUTS

- MSK_IINFITEM_MIO_NUM_CONTRA_CUTS

- MSK_IINFITEM_MIO_NUM_DISAGG_CUTS

- MSK_IINFITEM_MIO_NUM_FLOW_COVER_CUTS

- MSK_IINFITEM_MIO_NUM_GCD_CUTS

- MSK_IINFITEM_MIO_NUM_GUB_COVER_CUTS

- MSK_IINFITEM_MIO_NUM_KNAPSUR_COVER_CUTS

- MSK_IINFITEM_MIO_NUM_LATTICE_CUTS

- MSK_IINFITEM_MIO_NUM_LIFT_CUTS

- MSK_IINFITEM_MIO_NUM_OBJ_CUTS

- MSK_IINFITEM_MIO_NUM_PLAN_LOC_CUTS

- MSK_IINFITEM_SIM_NETWORK_DUAL_DEG_ITER

- MSK_IINFITEM_SIM_NETWORK_DUAL_HOTSTART

- MSK_IINFITEM_SIM_NETWORK_DUAL_HOTSTART_LU

- MSK_IINFITEM_SIM_NETWORK_DUAL_INF_ITER

- MSK_IINFITEM_SIM_NETWORK_DUAL_ITER

- MSK_IINFITEM_SIM_NETWORK_PRIMAL_DEG_ITER

- MSK_IINFITEM_SIM_NETWORK_PRIMAL_HOTSTART

- MSK_IINFITEM_SIM_NETWORK_PRIMAL_HOTSTART_LU

- MSK_IINFITEM_SIM_NETWORK_PRIMAL_INF_ITER

- MSK_IINFITEM_SIM_NETWORK_PRIMAL_ITER

- MSK_IINFITEM_SIM_PRIMAL_DUAL_DEG_ITER

- MSK_IINFITEM_SIM_PRIMAL_DUAL_HOTSTART

- MSK_IINFITEM_SIM_PRIMAL_DUAL_HOTSTART_LU

- MSK_IINFITEM_SIM_PRIMAL_DUAL_INF_ITER

- MSK_IINFITEM_SIM_PRIMAL_DUAL_ITER

- MSK_IINFITEM_SOL_INT_PROSTA

- MSK_IINFITEM_SOL_INT_SOLSTA

- MSK_IINFITEM_STO_NUM_A_CACHE_FLUSHES

- MSK_IINFITEM_STO_NUM_A_TRANSPOSES

- MSK_LIINFITEM_BI_CLEAN_PRIMAL_DUAL_DEG_ITER

- MSK_LIINFITEM_BI_CLEAN_PRIMAL_DUAL_ITER

- MSK_LIINFITEM_BI_CLEAN_PRIMAL_DUAL_SUB_ITER

- MSK_MIOMODE_LAZY

- MSK_OPTIMIZERTYPE_CONCURRENT

- MSK_OPTIMIZERTYPE_MIXED_INT_CONIC

- MSK_OPTIMIZERTYPE_NETWORK_PRIMAL_SIMPLEX

- MSK_OPTIMIZERTYPE_NONCONVEX

- MSK_OPTIMIZERTYPE_PRIMAL_DUAL_SIMPLEX

15.3 Response Codes

Added

- *"MSK_RES_ERR_CBF_DUPLICATE_PSDVAR"*

- *"MSK_RES_ERR_CBF_INVALID_PSDVAR_DIMENSION"*

- *"MSK_RES_ERR_CBF_TOO_FEW_PSDVAR"*

- *"MSK_RES_ERR_DUPLICATE_AIJ"*
- *"MSK_RES_ERR_FINAL_SOLUTION"*
- *"MSK_RES_ERR_JSON_DATA"*
- *"MSK_RES_ERR_JSON_FORMAT"*
- *"MSK_RES_ERR_JSON_MISSING_DATA"*
- *"MSK_RES_ERR_JSON_NUMBER_OVERFLOW"*
- *"MSK_RES_ERR_JSON_STRING"*
- *"MSK_RES_ERR_JSON_SYNTAX"*
- *"MSK_RES_ERR_LAU_INVALID_LOWER_TRIANGULAR_MATRIX"*
- *"MSK_RES_ERR_LAU_INVALID_SPARSE_SYMMETRIC_MATRIX"*
- *"MSK_RES_ERR_LAU_NOT_POSITIVE_DEFINITE"*
- *"MSK_RES_ERR_MIXED_CONIC_AND_NL"*
- *"MSK_RES_ERR_SERVER_CONNECT"*
- *"MSK_RES_ERR_SERVER_PROTOCOL"*
- *"MSK_RES_ERR_SERVER_STATUS"*
- *"MSK_RES_ERR_SERVER_TOKEN"*
- *"MSK_RES_ERR_SYM_MAT_HUGE"*
- *"MSK_RES_ERR_SYM_MAT_INVALID"*
- *"MSK_RES_ERR_TASK_WRITE"*
- *"MSK_RES_ERR_TOCONIC_CONSTR_NOT_CONIC"*
- *"MSK_RES_ERR_TOCONIC_CONSTR_Q_NOT_PSD"*
- *"MSK_RES_ERR_TOCONIC_CONSTRAINT_FX"*
- *"MSK_RES_ERR_TOCONIC_CONSTRAINT_RA"*
- *"MSK_RES_ERR_TOCONIC_OBJECTIVE_NOT_PSD"*
- *"MSK_RES_WRN_SYM_MAT_LARGE"*

Removed

- MSK_RES_ERR_AD_INVALID_OPERAND
- MSK_RES_ERR_AD_INVALID_OPERATOR
- MSK_RES_ERR_AD_MISSING_OPERAND
- MSK_RES_ERR_AD_MISSING_RETURN
- MSK_RES_ERR_CONCURRENT_OPTIMIZER
- MSK_RES_ERR_INV_CONIC_PROBLEM
- MSK_RES_ERR_INVALID_BRANCH_DIRECTION
- MSK_RES_ERR_INVALID_BRANCH_PRIORITY
- MSK_RES_ERR_INVALID_NETWORK_PROBLEM
- MSK_RES_ERR_MBT_INCOMPATIBLE
- MSK_RES_ERR_MBT_INVALID
- MSK_RES_ERR_MIO_NOT_LOADED

- MSK_RES_ERR_MIXED_PROBLEM

- MSK_RES_ERR_NO_DUAL_INFO_FOR_ITG_SOL

- MSK_RES_ERR_ORD_INVALID

- MSK_RES_ERR_ORD_INVALID_BRANCH_DIR

- MSK_RES_ERR_TOCONIC_CONVERSION_FAIL

- MSK_RES_ERR_TOO_MANY_CONCURRENT_TASKS

- MSK_RES_WRN_TOO_MANY_THREADS_CONCURRENT

- MSK_RES_ERR_MIXED_PROBLEM

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Enumerations

Functions

Parameters

Response codes

Structures

Types